

Problems on general probability rules, independence, conditional probability

About this problem set: Most of the problems here are “warm-up type” problems, intended to practice applying the appropriate definitions and rules. Many actuarial problems boil down to purely mathematical problems of this sort, after stripping off the disguises as a word problem. In fact, several of the problems here are essentially stripped-down versions of problems from the Actuarial Problem sets. (Try to spot those!)

1. Assuming A , B , C are mutually independent, with $P(A) = P(B) = P(C) = 0.1$, compute:
 - (a) $P(A \cup B)$
 - (b) $P(A \cup B \cup C)$
 - (c) $P(A \setminus (B \cup C))$

2. Given that $P(A) = 0.3$, $P(A|B) = 0.4$, and $P(B) = 0.5$, compute:
 - (a) $P(A \cap B)$
 - (b) $P(B|A)$
 - (c) $P(A'|B)$
 - (d) $P(A|B')$

3. Assume A and B are independent events with $P(A) = 0.2$ and $P(B) = 0.3$. Let C be the event that **at least one** of A or B occurs, and let D be the event that **exactly one** of A or B occurs.
 - (a) Find $P(C)$.
 - (b) Find $P(D)$.

- (c) Find $P(A|D)$ and $P(D|A)$.
- (d) Determine whether A and D are independent.
4. Given that $P(A \cup B) = 0.7$ and $P(A \cup B') = 0.9$, find $P(A)$.
5. Given that A and B are independent with $P(A) = 2P(B)$ and $P(A \cap B) = 0.15$, find $P(A' \cap B')$.
6. Given that A and B are independent with $P(A \cup B) = 0.8$ and $P(B') = 0.3$, find $P(A)$.
7. Given that $P(A) = 0.2$, $P(B) = 0.7$, and $P(A|B) = 0.15$, find $P(A' \cap B')$.
8. Given $P(A) = 0.6$, $P(B) = 0.7$, $P(C) = 0.8$, $P(A \cap B) = 0.3$, $P(A \cap C) = 0.4$, $P(B \cap C) = 0.5$, $P(A \cap B \cap C) = 0.2$, find $P(A \cap B' \cap C')$.