Worksheet: Common Errors in Writing up Induction Proofs

- **Undeclared/unquantified induction variable:** Starting out an induction step by saying “Suppose $P(n)$ is true for $n = k$”, without saying what $k$ is, what restrictions there have to be on $k$ (e.g., $k \geq 2$), would make this step incomplete. There are many examples of “false” induction proofs that are due to errors of this type.

- **Not stating, explicitly and precisely, the statement/formula that one seeks to prove.** For example, using language like “suppose it is true for $n = k$”, or “suppose $P(k)$”, without explicitly and precisely stating what “it” or “$P(k)$” means, would be wrong.

- **Induction hypothesis not referenced.** The most crucial part of any induction proof is the place where the induction hypothesis (e.g., the case $n = k$ of $P(n)$, or the cases $n = k$ and $n = k - 1$, etc) is being used. This is the place that can make or break an induction argument, and an error there can have disastrous consequences. **Always clearly state, at the appropriate place in the induction step, where the induction hypothesis is being used.**

- **Insufficient/inappropriate base cases:** The base case(s) must be sufficient to “kick-start” the chain reaction that the induction proof represents. For example, an induction in which the $k + 1$ case depends on the cases $k$ and $k - 1$ requires checking two consecutive $n$-values in the base step.

- **Base cases and induction step don’t match up:** The first case of the induction step (i.e., the first $k$-value for which the induction step is claimed) must match up with the base case(s), so as not to leave a gap in the chain reaction. There are numerous examples of “false” induction proofs that result from carelessness in this regard.

**Practice Problems**

In each of the following examples, something is wrong with the set-up or write-up of the induction proof. Find the error and try to correct it.

1. **Example 1.**
   - **Base step:** $n = 6$.
   - **Induction step:** Let $k \in \mathbb{N}$ be given and assume $(\ast)$ is true for $n = k$.

2. **Example 2.**
   - **Base step:** $n = 1$ and $n = 2$.
   - **Induction step:** Let $k \in \mathbb{N}$ with $k \geq 3$ be given and assume $(\ast)$ is true for $n = k$ and $n = k - 1$.

3. **Example 3.**
   - **Base step:** $n = 1$ and $n = 2$.
   - **Induction step:** Assume $(\ast)$ is true for $n = k$ and $n = k - 1$. Then ...

4. **Example 4.**
   - **Base step:** $n = 1$ and $n = 2$.
   - **Induction step:** Let $k \in \mathbb{N}$ be given and assume $(\ast)$ is true for $n = k$ and $n = k - 1$. 