1. **Definitions and theorems:** For the following questions an answer is sufficient: Just state the requested definition or theorem. Be sure to use correct notation, include any necessary quantifiers in the appropriate order, and connecting words (e.g., “such that”) if necessary.

(a) A relation \( R \) from a set \( S \) to a set \( T \) is ...

(b) Consider the congruence relation modulo 3 on the set \( S = \mathbb{Z} \). Write down explicitly all equivalence classes for this relation. Be sure to use correct notation.

(c) Let \( S \) be a set. A partition of \( S \) is a collection of nonempty subsets \( A_i \) of \( S \) satisfying certain properties. State these properties.

(d) State Fermat’s Little Theorem. Be sure to use correct notation and include any necessary conditions.

2. **Quickies:** Each of the following questions can be answered with a minimal amount of hand calculations, using the methods discussed in class. Calculators are not needed and not allowed, and answers arrived by brute force won’t count. Show work; an answer alone won’t count.

(a) Find the remainder of \( 2^{2010} \) upon division by 9.

(b) Find last decimal digit of \( 453^{101} \).

(c) Determine which (if any) of the numbers 2008, 2009, 2010 can be represented in the form \( 15x + 21y \), where \( x, y \in \mathbb{Z} \). Justify your answer (one or two sentences). (Note that this problem concerns only \( \text{existence} \) of such a representation; you do not need to \( \text{find} \) such a representation.)

(d) Define a relation on \( \mathbb{N} \) by \( x \sim y \iff (x, y) = 1 \). Determine whether this relation is (a) reflexive, (b) symmetric, (c) transitive. If the property holds, give a brief (one or two sentences) justification, if it does not hold, give a specific counterexample.

3. (a) Using the Euclidean algorithm, find the gcd of 13 and 17 and represent this gcd as a linear combination of 13 and 17 with integer coefficients.

(b) Find all integer solutions to the equation \( \frac{x}{13} + \frac{y}{17} = 1 \), or show that no such solutions exist.

4. (a) Determine, with brief explanation, whether or not there exist an integer \( x \) such that \( x^2 \equiv 3 \mod 5 \).

(b) Using only the definition of divisibility and/or congruences (i.e., without appealing to any of the theorems, propositions, properties, etc., about divisibility and congruences that you might know), give a careful proof of the following statement:

\[ \text{Let } a, b, c, d \in \mathbb{Z} \text{ and } m \in \mathbb{N}. \text{ If } a \equiv b \mod m \text{ and } c \equiv d \mod m, \text{ then } ac \equiv bd \mod m. \]

5. (a) Without using words of negation, state precisely what it means for a sequence \( \{a_n\} \) to be **not** bounded. Be sure to use correct logical notation and terminology and include any necessary quantifiers and connecting words.

(b) Using the definition of a limit, show that if \( a_n > 0 \) for all \( n \in \mathbb{N} \), \( \lim a_n = L \), and \( L > 0 \), then there exists a real number \( c > 0 \) such that \( a_n \geq c \) for all \( n \in \mathbb{N} \). (The proof should be done directly from the “\( \epsilon - N \)” definition of the limit of a sequence, and **not** use any of the properties, lemmas, propositions, etc. of limits established in the book or in class.)