#1 Properties of relations. In each of the following problems, a set $S$ and a relation on $S$, denoted by $\sim$, are defined. Determine whether the given relation is reflexive (R), symmetric (S), transitive (T), and an equivalence relation (E). For each of the properties (R), (S), (T), either give a proof, or a specific counterexample. For (E) an answer, based on the answers to (R), (S), (T), is sufficient. The proofs required are quite short—one or two lines usually are enough for symmetric and reflexive properties, and a bit more for the reflexive property—but they must written up with the usual care, using the proper definitions (e.g., for divisibility).

(a) $S = \{1, 2, \ldots, 16\}$. $x \sim y \iff x + y = 17$.
(b) $S = \mathbb{N}$. $x \sim y \iff x \mid y$.
(c) $S = \mathbb{R}$. $x \sim y \iff x - y \in \mathbb{Z}$.
(d) $S = \mathbb{R} - \{0\}$. $x \sim y \iff \text{there exists } q \in \mathbb{Q} \text{ such that } x = qy$.

#2 Equivalence classes. For those relations in Problem 1 that are equivalence relations, determine explicitly the equivalence class of the element 3, i.e., the set $E_3$ in the notation from the Relations handout. Explain briefly! (The point of this exercise is to determine these sets explicitly, i.e., not by using general notations such as $\{x \in \mathbb{N} : x \sim 1\}$.

For example, for the congruence relation modulo 2 on the set $S = \mathbb{Z}$, the equivalence class $E_3$ is the set of odd integers.

Final HW Problem: The following is a problem from the text that is a particularly instructive exercise in that it involves concepts and methods from three different chapters: Equivalence relations (Chapter 7), complex logical statements with multiple quantifiers (Chapter 2), and proofs involving functions from $\mathbb{R}$ to $\mathbb{R}$ (Chapters 1/4). All of these are things you will need to know on the Final Exam, so this exercise will help you prepare for the final. Pay attention to the write-up of your proof, especially that of transitivity, which requires a careful, step-by-step argument, in the correct logical order.

#3 7.14. Let $f$ be a given function from $\mathbb{R}$ to $\mathbb{R}$ and define a relation on the set of all functions from $\mathbb{R}$ to $\mathbb{R}$ as follows:

$$g \sim h \iff (\exists c, a \in \mathbb{R}) (\forall x \geq a) \left( |g(x) - h(x)| \leq c|f(x)| \right).$$

Give a careful proof that this relation is an equivalence relation.