1. Evaluate the integral \( I = \int_0^\infty e^{-\frac{1}{2}x^2} \, dx \) by considering its square, \( I \cdot I \), and evaluating the latter as a double integral. (Note that the range of integration is restricted to positive values of \( x \).)

2. Suppose \( X \) and \( Y \) are random variables with joint density function

\[
f(x, y) = \begin{cases} 
Ce^{-x-4y} & \text{if } x \geq 0 \text{ and } y \geq 0, \\
0 & \text{otherwise,}
\end{cases}
\]

where \( C \) is a constant.

(a) Determine the constant \( C \).

(b) Find the probability \( P(X + Y \leq 1) \).

3. Consider the transformation of variables in \( \mathbb{R}^3 \) defined by

\[
x = u^2, \quad y = v^3, \quad z = w^4.
\]

(a) Compute the Jacobian determinant \( \frac{\partial(x, y, z)}{\partial(u, v, w)} \) of the above transformation.

(b) Using the above transformation, set up, but do not evaluate, a triple integral in the variables \( u, v, w \), that gives the volume of the region (in the first octant) that is bounded by the surface \( x^{1/2} + y^{1/3} + z^{1/4} = 1 \) and the coordinate planes. Your answer should be of the form \( V = \int \ast \ast \ast \, du \, dv \, dw \), with appropriate expressions in place of the 7 asterisks.

4. Quickies. The following problems are independent of each other.

(a) Convert the equation \( \rho = \cos \phi \) into an equation in rectangular coordinates, simplifying as much as possible. (In particular, the equation should not involve inverse trig functions.)

(b) Express the vector line integral \( \int_C \vec{F} \cdot d\vec{r} \) as a line integral with respect to arclength \( ds \), i.e., in the form \( \int_a^b \ast \ast \ast \, ds \), with appropriate expressions in place of the asterisks. Be sure to use correct notation. (As usual, \( C \) denotes a smooth curve parametrized by \( \vec{r}(t), a \leq t \leq b \), and \( \vec{F} = \langle P, Q \rangle \) a vector field in \( \mathbb{R}^2 \) whose component functions \( P \) and \( Q \) have continuous partial derivatives.)

(c) Find the volume of the image of the \( n \)-dimensional unit cube under the transformation \( \vec{x} = T(\vec{u}) \) from \( \mathbb{R}^n \) to \( \mathbb{R}^n \) defined by

\[
x_1 = 2u_1, \\
x_2 = 2u_2 - u_1 \\
x_3 = 2u_3 - u_2, \\
\ldots \\
x_n = 2u_n - u_{n-1},
\]

Be sure to show and explain your work.

5. Let \( a, b, c, d \) be real numbers, and let \( \vec{F} \) be the vector field in \( \mathbb{R}^2 \) defined by

\[
\vec{F}(x, y) = \langle ax + by, cx + dy \rangle.
\]

(a) Find the work performed under the field \( \vec{F} \) when moving an object along a straight line path from \((0, 0)\) to \((2, 3)\).

(b) Under what conditions on the values of \( a, b, c, d \) does the field \( \vec{F} \) have a potential? Justify your answer.

(c) Using a systematic approach (not by guessing or by trial and error), find a potential for \( \vec{F} \) in those cases in which a potential exists (i.e., the cases that meet the conditions of the previous part).
6. Setup of integrals. Let $E$ denote the solid region in $\mathbb{R}^3$ defined by

$$
\begin{align*}
-2 & \leq x \leq 2, \\
0 & \leq y \leq \sqrt{4-x^2}, \\
\sqrt{x^2+y^2} & \leq z \leq \sqrt{8-x^2-y^2}.
\end{align*}
$$

For each of the following problems, set up, but do not evaluate, an iterated integral of the requested type in the coordinate system specified. Your answer should be in the form

$$\int \int \int \, d \, d \, d$$

or

$$\int \int \int \, d \, d \, d$$

with appropriate expressions in place of the asterisks. (Hint: You may want to sketch the appropriate regions to determine the correct integration limits.)

(a) The volume of $E$ as a double integral in rectangular coordinates.
(b) The volume of $E$ as a triple integral in cylindrical coordinates.
(c) The volume of $E$ as a triple integral in spherical coordinates.