Math 571: Model Theory  
Spring Semester 2008  
Prof. Ward Henson  

Problem Set 3  
Due in class Monday, March 10  

There are four problems (equally weighted) and you should do all of them. To earn full credit requires a careful writeup of each problem, taking care to justify everything you claim and to explain your ideas clearly. Do not look up solutions in textbooks or other sources, and make sure your submitted solutions are your own work.

3.1 Problem. Let \( L \) be the language whose only nonlogical symbol is a binary predicate symbol \(<\). Let \( T \) be the \( L \)-theory of discrete linear orderings with a least element but no maximum element.
- Using methods similar to those used in Example 5.6, construct carefully a conservative extension of \( T \) that admits QE. (Note that \( T \) itself does not admit QE; for example, the least element of a model of \( T \) is definable but not by a quantifier-free formula.)
- Use the first item to show that \( T \) is complete, so \( T = \text{Th}(\mathbb{N}, <) \).

3.2 Problem. Use the preceding Problem to show that the model \((\mathbb{N}, <)\) is minimal, but that its theory is not strongly minimal.

3.3 Problem. Let \( L \) be any first order language. Let \( A, B \) be \( L \)-structures such that \( A \preceq B \).
- Show that for any \( X \subseteq A \) one has \( \acl_A(X) = \acl_B(X) \).

3.4 Problem. Let \( T \) be a strongly minimal \( L \)-theory and let \( \kappa \) be an infinite cardinal. Let \( A \) be an infinite model of \( T \).
- Show that \( A \) is \( \kappa \)-saturated iff the dimension of \( A \) in the sense of Section 10 is \( \geq \kappa \).
- If \( \kappa > \text{card}(L) \), show that \( T \) has a \( \kappa \)-saturated model whose cardinality is equal to \( \kappa \).

Note: If \( T \) is complete, by Theorem 10.8(3), the model described in the previous item is unique up to isomorphism. See also Theorem 12.6 for a stronger uniqueness result.