1.1 Problem. Let $L$ be the language whose only nonlogical symbol is a binary predicate symbol $E$. Let $K$ be the class of $L$-structures in which $E$ is interpreted by an equivalence relation that has precisely one equivalence class of cardinality $n$ for each positive integer $n$. Let $T$ be the theory of $K$.

- Show that $K$ is the class of all models of $T$.
- Show that $T$ does not admit QE.
- In which cardinalities $\kappa$ is $T$ $\kappa$-categorical? ($T$ is $\kappa$-categorical iff $T$ has a model of cardinality $\kappa$ and any two models of $T$ that have cardinality $\kappa$ are isomorphic.)

1.2 Problem. Let $L$ be the language whose only nonlogical symbols are a binary function symbol $+$ and a constant symbol 0. For each field $F$ let $V_2(F)$ be the $L$-structure whose underlying set is \{$(a, b) \mid a, b \in F$\}, whose interpretation of $+$ is given by $(a, b) + (a', b') = (a + a', b + b')$, for any $a, b, a', b' \in F$, and whose interpretation of 0 is $(0, 0)$. Let $K$ be the class of all $L$-structures that are isomorphic to $V_2(F)$ for some field $F$.

- Show that $K$ is closed under the ultraproduct construction.

1.3 Problem. Let $L$ be the language whose only nonlogical symbol is the unary predicate symbol $P$. Let $K$ be the class of all $L$-structures $A$ such that $P^A$ and $A \setminus P^A$ are both infinite. Let $T$ be the theory of the class $K$.

- Classify the models of $T$ up to isomorphism.
- Use local isomorphisms to show that $T$ admits QE and is complete. (See the proof of Example 3.15.)

1.4 Problem. Consider the theory DLO of dense linear orderings without end points. Let $A = (A, <)$ be a model of DLO.

- Characterize the functions $f: A \to A$ that are $A$-definable in $A$.

(A function $f: A \to A$ is $A$-definable in $A$ if there exists a formula $\varphi(x, y, z_1, \ldots, z_n)$ in the language of DLO and elements $e_1, \ldots, e_n$ of $A$ such that $b = f(a) \iff A \models \varphi[a, b, e_1, \ldots, e_n]$ for all $a, b \in A$. By Example 3.15, $\varphi$ can be taken to be a quantifier-free formula.)