1. (3 points) Let $L_1$ be the first-order language with a single non-logical symbol $P$, where $P$ is a binary predicate symbol. Let $A$ be a structure for $L_1$ and $h: |A| \rightarrow |A|$ a function.

**Define:** $h$ is an automorphism of $A$ if:
- $h$ is a bijection from $|A|$ onto $|A|$;
- for all $a, b \in |A|$, we have $P^A(a, b) \iff P^A(h(a), h(b))$.

2. (4 points) Let $L_2$ be the first-order language with a single non-logical symbol $Q$, where $Q$ is a unary predicate symbol. Let $B$ be the structure for $L_2$ with $|B| = \mathbb{N}$ and $Q^B = \{0, 1, 2\}$.

**Characterize the automorphisms of $B$.** (Do not just give the definition.)

**Ans:**
- The restriction of $h$ to $Q^B$ is one of the 6 possible permutations of $\{0, 1, 2\}$ and the restriction of $h$ to $\mathbb{N} \setminus Q^B$ is an arbitrary permutation of that set.
3 (3 points) Give a prenex formula that is logically equivalent to:

\[(\forall x \, Qx \rightarrow \exists x \,(P_x \rightarrow \forall y \, R_{xy}))\]

\[\exists x \, \exists z \, \forall y \,(Q_x \rightarrow (P_z \rightarrow R_{zy}))\]

And

(For example)