1 (3 points) Let $A$ be an interpretation (structure) for a first-order language $L$ and let $s: V \to |A|$, where $V$ is the set of variables of $L$. Complete the following equivalence so it states the quantifier clause of the inductive definition of the satisfaction relation: for any wff $\varphi$ of $L$ and $x$ in $V$:

\[ \models_A \forall x \varphi [s] \iff \left[ \begin{array}{l}
\text{For every } d \in |A| \\
\models_A \varphi [s(x|d)]
\end{array} \right] \]

2 (7 points) Give a careful proof, using the inductive definition of satisfaction, of the following implication, for any wff $\alpha$ and variable $x$:

\[ \forall x \alpha \models \alpha \]

Proof:

Let $A$ be any interpretation and $s: V \to |A|$ any function ($V =$ variables of the language) we assume $\models_A \forall x \alpha [s]$ and need to prove $\models_A \alpha [s]$. This follows from in the equivalence above by taking $d = s(x)$. (Note that $s(x|s(x))$ equals $s$.)