Problem 1. (20 points) Consider the problem of approximating the definite integral
\[ \int_{0}^{2} e^{-x^2} \, dx. \]
You are free to choose which method of approximation you want to consider: left sum, right sum, trapezoid sum, or midpoint sum. Let \( n \) be the number of small intervals into which \([0,2]\) is subdivided for your approximation. Find \( n \) so that your method of approximation is guaranteed to estimate the integral within an error of 0.05. (You need not actually carry out the approximation; just tell which method and how many subdivisions to use. As must be done for all problems, justify that your answer meets the given requirements.)

Problem 2. (20 points) Consider a bucket hanging at the end of a 50 foot long rope; the rope weighs 0.25 pounds per foot and the bucket weighs 100 pounds. How much work is done when the rope is wound up at the top enough to raise the bucket 20 feet?

Problem 3. (20 points) Find the antiderivatives of the function
\[ f(x) = (x + 2)e^{3x}. \]

Problem 4. (20 points) Determine whether or not the following improper integral converges:
\[ \int_{0}^{1} \frac{1}{x^2 - 2x} \, dx \]

Problem 5. (20 points) Show that the following series converges; give a full justification with all details.
\[ \sum_{n=1}^{\infty} \frac{1}{n^4 + 5n + 1} \]
**Problem 6.** (20 points) Determine the radius of convergence of the following power series:

\[
\sum_{n=1}^{\infty} \frac{x^{2n+1}}{2^n (n + 1)}
\]

**Problem 7.** (20 points) Find a solution \( y = f(x) \) of the following initial value problem:

\[
y' = y \cos^3(x) \quad y(0) = 2
\]

**Problem 8.** (20 points) Consider the graph of the following equation in polar coordinates:

\[
r = 2 \cos(2\theta)
\]

(a) sketch this graph.

(b) calculate the area of one of the four leaves in this graph.

**Problem 9.** (20 points) Find the equation of the ellipse that has its foci at (4,5) and (4,-1) and has a minor axis of total length 8. What is the length of the major axis of this ellipse?

**Problem 10.** (20 points) Consider the set of all points \((x,y)\) in the \(xy\)-plane satisfying the equation

\[
9x^2 - y^2 + 36x + 4y + 23 = 0.
\]

(a) determine what kind of conic section this equation describes.

(b) determine (as appropriate) the following data for this conic section: foci, vertices, asymptotes.