Math 181: Activity
Planning a Road Trip
1/29/18

1. With your group, pick 6 cities that you can travel between by car. Fill out the following table with your cities and the time it takes to travel between them. (Feel free to use your phone or computer to look up travel times.)

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2. Draw a weighted graph that contains the information from your table.

3. Apply the nearest neighbor algorithm to find a Hamiltonian cycle on your graph.

4. Apply the sorted edges algorithm to find a Hamiltonian cycle on your graph.
5. Pick a subset of four vertices and apply the method of trees to find the shortest cycle visiting all four of those vertices.

6. Apply Kruskal’s Algorithm to find the minimum cost spanning tree on your graph (assuming that the travel times are the costs for each edge).
7. Let’s think about why it is important to have these algorithms for solving the Traveling Salesperson Problem.

(a) How many different Hamiltonian cycles are there in a complete graph with 4 vertices?

(b) How many different Hamiltonian cycles are there in a complete graph with 5 vertices?

(c) Now think about how many Hamiltonian cycles there are with \( n \) vertices.
   
i. Since we are looking at cycles, it does not matter which vertex we start with. How many ways are there to choose the second vertex in the cycle?

   ii. How many ways are there to choose the third vertex in the cycle? Fourth?

   iii. To find the number of ways to create full cycles, we use something called the multiplication principle. It states that if there are \( a \) ways of making a first choice and \( b \) ways of making a second choice, there are \( ab \) ways to make the two choices. Using this principle, how many Hamiltonian cycles are there in a complete graph on \( n \) vertices?

(d) Draw a picture that shows why the multiplication principle is true. (Hint: Where can you see it in the method of trees?)