Math 181 – Exam 1
Spring 2018
100 points possible

- Make sure to put your name on your exam!

- There are a total of 9 questions.

- Do not the exam booklet until you are told to start.

- Please write clearly and justify your answers.

- No calculators.

- No materials other than a pen, pencil, and eraser.

- Stop working and close the exam when time is called.

- Please note that the questions are not necessarily in order of difficulty.

- Good luck. You have 50 minutes to complete the exam.
1. Answer the questions about the graph below.

(a) How many vertices does the graph have?
(b) How many edges does the graph have?
(c) List the degree of each vertex.
(d) Is the graph 2-colorable?
(e) Is the graph 3-colorable?
(f) Is the graph 4-colorable?
(g) What is the chromatic number of the graph?
(h) Is the graph connected?
2. 
   (a) Draw a connected graph on six vertices where each vertex has degree 2.

   (b) How many edges does your graph in part (a) have?

   (c) Draw a connected graph on six vertices where each vertex has degree 5.

   (d) How many edges does your graph in part (c) have?

3. True or False.
   (a) If a graph has an Euler circuit, then it must be connected.

   (b) If a graph is connected, then it must have an Euler circuit.

   (c) If a graph is complete, then it must have a Hamiltonian cycle.

   (d) If a graph has a Hamiltonian cycle, then it must have an Euler circuit.

   (e) If a graph has an Euler circuit, then it must have a Hamiltonian cycle.
4. Determine whether the following graphs have an Euler circuit. If yes, then number the edges in the order an Euler circuit uses them. If no, then explain why not.

![Graph (a)](image1)

(a)  

![Graph (b)](image2)

(b)
The table below shows the mileage between four cities in Illinois.

<table>
<thead>
<tr>
<th></th>
<th>Champaign</th>
<th>Eldorado</th>
<th>Geneva</th>
<th>Paris</th>
</tr>
</thead>
<tbody>
<tr>
<td>Champaign</td>
<td>–</td>
<td>177</td>
<td>160</td>
<td>67</td>
</tr>
<tr>
<td>Eldorado</td>
<td>177</td>
<td>–</td>
<td>352</td>
<td>144</td>
</tr>
<tr>
<td>Geneva</td>
<td>160</td>
<td>352</td>
<td>–</td>
<td>198</td>
</tr>
<tr>
<td>Paris</td>
<td>67</td>
<td>144</td>
<td>198</td>
<td>–</td>
</tr>
</tbody>
</table>

(a) Represent this information by drawing a weighted graph on four vertices.

(b) Use the Nearest Neighbor Algorithm starting at Champaign to find a Hamiltonian cycle.

(c) Use the Nearest Neighbor Algorithm starting at Geneva to find a Hamiltonian cycle.

(d) Use the Sorted Edges Algorithm to find a Hamiltonian cycle.
6. Use the Method of Trees to find all possible Hamiltonian cycles in the graph below.

What is the weight of the shortest Hamiltonian cycle?
7. Kruskal’s Algorithm
   
   (a) Use Kruskal’s Algorithm to find a minimum weight spanning tree in the graph below.

   (b) What is the weight of the spanning tree you found?

   (c) Is this spanning tree optimal or can we find a spanning tree with less weight?
8. Find the chromatic number for each graph below. Justify your answer. Color the graph with that many colors.

(a) 

(b)
9. Matchings

(a) Does the following graph have a perfect matching? If yes, show the matching; if no, explain why not.

(b) Does the following graph have a perfect matching? If yes, show the matching; if no, explain why not.