Math 181: Activity
Manipulating Elections

1. Consider the following preference lists.

<table>
<thead>
<tr>
<th>Voter 1</th>
<th>Voter 2</th>
<th>Voter 3</th>
<th>Voter 4</th>
<th>Voter 5</th>
<th>Voter 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>A</td>
<td>B</td>
<td>A</td>
<td>E</td>
<td>A</td>
</tr>
<tr>
<td>2nd</td>
<td>B</td>
<td>C</td>
<td>B</td>
<td>D</td>
<td>B</td>
</tr>
<tr>
<td>3rd</td>
<td>C</td>
<td>A</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>4th</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>B</td>
<td>D</td>
</tr>
<tr>
<td>5th</td>
<td>E</td>
<td>E</td>
<td>E</td>
<td>A</td>
<td>E</td>
</tr>
</tbody>
</table>

(a) Calculate the winner using the Borda count.

A: \(3(4) + 1(2) + 2(0) = 14\)

B: \(3+4+3+1+3+1 = 15\)

C: \(2+3+2+2+2+2 = 13\)

D: \(1+1+1+3+1+3 = 10\)

E: \(0+0+0+4+0+4 = 8\)

(b) What is the outcome using the Borda count if only Voters 3-6 voted?

A: \(4+0+4+0 = 8\)

B: \(3+1+3+1 = 8\)

C: \(2+2+2+2 = 8\)

D: \(1+2+1+2 = 6\)

E: \(0+4+0+4 = 8\)

4-way tie between A, B, C & E

(c) As voter 1, can you manipulate the election (change your ranking) so that Candidate A wins?

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<tr>
<th>Voter 1</th>
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<th>Voter 5</th>
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<td>D</td>
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<td>D</td>
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<tr>
<td>5th</td>
<td>B</td>
<td>E</td>
<td>E</td>
<td>A</td>
<td>E</td>
</tr>
</tbody>
</table>

A: 14

B: 12

C: 13

D: 10

E: 11

A wins
2. Consider the following preference lists.

<table>
<thead>
<tr>
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<th>Voter 1</th>
<th>Voter 2</th>
<th>Voter 3</th>
</tr>
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<tbody>
<tr>
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<td>D</td>
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<td>C</td>
</tr>
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<td>4th</td>
<td>C</td>
<td>D</td>
<td>A</td>
</tr>
</tbody>
</table>

(a) Give an agenda where A wins sequential pairwise voting.

\[
\text{CDBA}
\]

(b) Give an agenda where B wins sequential pairwise voting.

\[
\text{CADB}
\]

(c) Give an agenda where C wins sequential pairwise voting.

\[
\text{BADC}
\]

(d) Give an agenda where D wins sequential pairwise voting.

\[
\text{BACD}
\]

3. Consider the following set of preference lists.

<table>
<thead>
<tr>
<th></th>
<th>(4)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>A</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>2nd</td>
<td>B</td>
<td>A</td>
<td>C</td>
</tr>
<tr>
<td>3rd</td>
<td>C</td>
<td>B</td>
<td>A</td>
</tr>
</tbody>
</table>

(a) Who wins this election using the Hare System?

\[
\text{A}
\]

R1: \[A:4 \quad B:4 \quad C:3\]

R2: \[A:7 \quad B:4\]

(b) Now imagine that one of the voters with the ballot in the right column changed their vote to \(C > B > A\). Who wins now?

C

R1: \[A:4 \quad C:4 \quad B:3\]

R2: \[A:4 \quad C:7\]

(c) What does this tell us about the Hare System? What about two-round plurality?

\[
\text{Can be manipulated}
\]

With 3 candidates, the Hare system and two-round plurality are the same \(\Rightarrow\) two-round plurality can also be manipulated.
4. The Chair's Paradox

In this example we will vote using the plurality method with one caveat: if there is a tie, the candidate the chair voted for wins. This is common for votes in small committees. Consider the following set of preference lists.

<table>
<thead>
<tr>
<th></th>
<th>chair</th>
<th>Member 1</th>
<th>Member 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>2nd</td>
<td>B</td>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td>3rd</td>
<td>C</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

(a) Who wins this election?

\[ A: \text{3-way tie} \Rightarrow \text{chair's candidate wins} \]

(b) How can Member 1 manipulate their vote to get a more preferred result?

\[ \text{vote} \begin{array}{c} \text{C} \\ \text{B} \\ \text{A} \end{array} \Rightarrow \text{C wins} \]

(c) Is there any way for the chair to manipulate their vote to get a more preferred result?

\[ \text{read sec 10.5 for more explanation} \]

No: if no one else manipulates, chair gets 1st choice

\[ \text{if M2 manipulates, nothing chair can do.} \]

5. Consider a voting system where a candidate is among the winners if it receives at least one first place vote. Is this system manipulable? Why or why not?

\[ \text{No. if you mark candidate as 1st choice it will win. No way to do better than your 1st choice.} \]

6. Consider the voting system for two candidates and three voters in which the candidate receiving an odd number of first-place votes wins. Is the system manipulable? Why or why not?

\[ \text{yes consider} \]

\[ \begin{array}{ccc} A & A & B \\ B & B & A \end{array} \rightarrow \text{B wins} \]

\[ \triangledown \]

this voter can manipulate the election by voting for B instead of A.