NAME: ____________________________________________
(Please Print)

Circle your discussion  11am  12pm  1pm  2pm  3pm

DIRECTIONS:

• Sit in the seat indicated. DO NOT OPEN YOUR EXAM UNTIL YOU ARE TOLD TO DO SO!

• Each exam is worth 200 points and has a total of 13 pages.

• No work means no points.

• This is a closed book, closed notes exam. No calculators allowed.

• No hats or dark sunglasses.

• No cell phones. Turn them OFF now. If you are seen with a cell phone in hand during the exam or if your cell phone is heard, it will be considered cheating and you will be asked to leave.

• No other electronic devices – MP3 players, PDAs, etc. Same rules as phones.

• If you have a question, raise your hand and a proctor will come to you. Once you stand up, you are done with the exam. You will not be permitted to leave the room and return during the exam.

• Be sure to write your proper name CLEARLY and circle the discussion section for which you are registered.

• If you finish early, quietly and respectfully get up and hand in your exam. You need to show a picture ID with a clear picture when you turn in your exam.

• When time is up, you will be instructed to put down your writing utensil and close your exam. Anyone seen continuing to write after this announcement will have their exam marked and will lose all points for the page on which they are writing.

• When you are done, enjoy your summer break.
**SCORES:**

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Formulas:

\[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \]

\[ \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \]

\[ \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} \]

\[ \sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2 + 3n - 1)}{30} \]

\[ \sum_{i=1}^{n} i^5 = \frac{n^2(n+1)^2(2n^2 + 2n - 1)}{12} \]
Problem 1. (20 points)
Compute derivatives of the following functions.

a) $f(x) = \frac{e^{2x} - \sin^{-1} x}{\ln x}$.

b) $g(t) = te^{(4t^2 - \tan t + \ln t)}$. 
c) \( h(s) = \ln(\sec x + \tan x) \).

d) \( j(w) = w^w \). [Hint: Use logarithmic differentiation.]
Problem 2. (7 points)
The Mean Value Theorem begins “Suppose $f$ is continuous on $[a,b]$ and differentiable on $(a,b) \ldots$”

a) State the conclusion to the Mean Value Theorem.

b) Draw a picture that illustrates the conclusion to the Mean Value Theorem.
Problem 3. (12 points)
Given the graph of $f''(x)$ [on the left] and $f'(x)$ [on the right],

a) Determine where $f(x)$ is increasing and decreasing.

b) Determine where $f(x)$ is concave up and concave down.

c) Locate and classify the extrema of $f$. 
Problem 4. (9 points)
a) State the formal, precise definition of the statement
\[ \lim_{x \to a} f(x) = L. \]

b) Draw a picture that illustrates this definition. Be sure to label important aspects of your picture.
Problem 5. (20 points)
Compute the following limits.

a) \( \lim_{x \to \infty} \frac{\sqrt{x + 1}}{\sqrt{x} - 1} \).

b) \( \lim_{x \to 2} \frac{x^2 + 2x - 8}{x^2 - 4} \).
c) \( \lim_{x \to 2} \frac{x^2 + 4}{x - 2} \)

d) \( \lim_{x \to 0} x^2 \cos \left( \frac{1}{x} \right) \).
Problem 6. (8 points)
State any three of the four Limit Laws. Be sure to state any needed hypotheses.
**Problem 7. (10 points)**

a) State the definition of the derivative $f'(x)$.

b) From the definition of the derivative, calculate the derivative of $f(x) = \frac{1}{3x + 4}$.
Problem 8. (5 points)
Find the area between the curves $y = \sqrt{x}$ and $y = x^3$ on $[0, 1]$.

Problem 9. (6 points)
Set up an expression for the arclength but DO NOT EVALUATE!

a) $y = 2x - x^2$ on $1 \leq x \leq 3$.

Set up an expression for the surface area but DO NOT EVALUATE!

b) $y = \sin x$, $0 \leq x \leq \pi$ revolved around the $x$-axis.
Problem 10. (10 points)
The equivalent capacitance of a sequence of capacitors in series is given by
\[ L_{eq} = \left( \frac{1}{L_1} + \frac{1}{L_2} \right)^{-1} \]
where \( L_1 \) and \( L_2 \) are the capacitances of the capacitors.

You have enough material to make 10 \( \mu \text{F} \) (a unit of capacitance) worth of capacitors. If you make two resistors, what is the maximum equivalent capacitance you can create?
Problem 11. (10 points)
a) State the definition of the area under the curve \( y = f(x) \) on \([a,b]\).

b) From the Riemann Sum definition of (signed) area under the curve, calculate the area under the curve \( y = 2x^2 + 3x \) on \([0,1]\).
Problem 12. (5 points)
State one version of the Fundamental Theorem of Calculus with all hypotheses.

Problem 13. (4 points)
Ignoring air resistance, and using that the acceleration due to gravity is \(-32\,\text{ft}\,s^{-2}\),
an object is dropped from a height of 64 ft.
a) Find when the object hits the ground.

b) Find the velocity of the object when it hits the ground.
Problem 14. (9 points) A scientist measures the following data,

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<th>0</th>
<th>0.1</th>
<th>0.2</th>
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<tr>
<td>f(t)</td>
<td>2.3</td>
<td>3.4</td>
<td>1.2</td>
<td>2.2</td>
<td>0.1</td>
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<td>3.1</td>
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If \( f(t) \) measures a current in Amperes, the signed area under the curve would represent a total electric charge in Coulombs that has passed by the measurement point.

a) Approximate the total charge by using Simpson’s Rule.

b) Approximate the total charge by using left endpoints.

c) Briefly explain why Simpson’s Rule should give a better approximation than the left end point approximation. [Do not write more than one sentence.]
Problem 15. (7 points)
The Second Derivative Test begins “Suppose that $f$ is continuous on the interval $(a, b)$ and $f'(c) = 0$ for some $c \in (a, b) \ldots”$

a) State the conclusion of the Second Derivative test.

b) Draw a picture that explains why the Second Derivative Test makes sense. Also write a complete sentence explaining your picture.
Problem 16. (9 points)

Consider the function

\[ f(x) = \begin{cases} 
4 \tan^{-1} x - e^{2x} + 7 & x < 0 \\
5 & x = 0 \\
2 \ln(1 + x) + \frac{1}{6} x^3 + 4 \cos x + 2 & x > 0 
\end{cases} \]

a) Determine \( \lim_{x \to 0^-} f(x) \) and \( \lim_{x \to 0^+} f(x) \) and \( \lim_{x \to 0} f(x) \).

b) Determine whether \( f \) is continuous at zero or not.
Problem 17. (20 points)
Compute the following.

a) \[ \int_{4}^{47} \left( \frac{(\ln t)^2}{t} \right) dt. \]

b) \[ \int e^{\tan^{-1} r} \frac{\tan^{-1} r}{1 + r^2} dr. \]
c) \[ \int_{0}^{1} \left( \frac{1 + x}{1 + x^2} \right) \, dx. \]

d) \[ \int \left( \cot y \csc y - 5y^3 + \frac{4}{y} \right) \, dy \]
Problem 18. (5 points)
Use one iteration of Newton’s Method or use the linear approximation $L(x)$ to approximate $\sqrt[3]{28}$.

Problem 19. (5 points)
A snowball, located in a very hot and unpleasant place, is melting. If the snowball is always a sphere, how fast is the radius changing when the snowball is melting at a rate of $4 \frac{cm^3}{s}$ and the radius is exactly $12 cm$?
Problem 20. (9 points)
The function $G(x) = \int_{1}^{x} \cos(e^{2t}) \, dt$,

a) Calculate the second derivative of $G(x)$.

b) Determine the concavity of $G$ at $x = \frac{1}{4} \ln(\pi)$. 
Problem 21. (15 points)
Let $R$ be the region bounded by $y = x^2$, $y = 0$ and $x = 1$. Compute the volume of the solid formed by revolving $R$ about the given line.

a) The $x$–axis.

b) The $y$–axis.
c) The line $x = -1$. 
Bonus Question: (6 points) - Don’t spend time on this unless you have finished all the other questions!

Calculate

\[ \int (x^x \ln x + x^x) \, dx \]