Math 285 Quiz 1
Show all your work to get a full credit.

Problem 1 Solve the given IVP.
\[
\frac{dy}{dx} = 2y - 1, \quad y(0) = 1.
\]

solution
Note that this is a separable equation. So write the above equaiton as \( \frac{dy}{2y - 1} = dx \).

\[
\int \frac{dy}{2y - 1}
\]

\[
\frac{1}{2} \ln|2y - 1| = x + C.
\]

\[
\ln|2y - 1| = 2x + C.
\]

\[
|2y - 1| =
\]

\[
2y - 1 = Ce^{2x}.
\]

So, \( y = \frac{1}{2} (Ce^{2x} + 1) \).

Since \( y(0) = 1 \), \( 1 = \frac{1}{2} (C + 1) \). Hence \( C = 1 \).

Thus, \( y = \frac{1}{2} (e^{2x} + 1) \).

Note: Here, I used a constant \( C \) abusively.

Problem 2 Determine whether the existence and uniqueness of a solution of the following IVP are guaranteed.

\[
\frac{dy}{dx} = \frac{x-1}{y}, \quad y(0) = 1.
\]

solution
Since \( f(x,y) = \frac{x-1}{y} \) is continuous around \((0,1)\), the existence of the solution is guaranteed.

Since \( \frac{\partial f}{\partial y} = -\frac{x-1}{y^2} \) is also continuous around \((0,1)\), the uniqueness of a solution is guaranteed.