8. From Equation (40) with \( k = 1 \) and \( L = 2 \) we get

\[
u(x, t) = a_0/2 + \sum a_n e^{-(n^2 \pi^2 t/4)} \cos n\pi x/2.
\]

But

\[10 \cos \pi x \cos 3\pi x = 5 \cos 2\pi x + 5 \cos 4\pi x.
\]

Hence we choose \( b_4 = b_8 = 5 \) and \( b_n = 0 \) otherwise to get

\[u(x, t) = 5 e^{-4\pi^2 t} \cos 2\pi x + 5 e^{-16\pi^2 t} \cos 4\pi x.
\]

10. From Equation (31) with \( k = 1/5 \) and \( L = 10 \) we get

\[u(x, t) = \sum b_n e^{-(n^2 \pi^2 t/500)} \sin n\pi x/10.
\]

On the basis of Equation (16) in Section 9.3 we choose \( b_n = 80(-1)^{n+1}/\pi n \) for \( n = 1, 2, 3, \ldots \).

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Section 9.6

4. Here \( a = 1/2 \) and \( L = 2 \), so \( n\pi x/L = n\pi x/2 \) and \( n\pi at/L = n\pi a/4 \). To satisfy the condition

\[y(x, 0) = (1/5) \sin \pi x \cos \pi x = (1/10) \sin 2\pi x = (1/10) \sin 4\pi x/2,
\]

we choose \( A_4 = 1/10 \) and \( A_n = 0 \) for \( n \neq 4 \). To satisfy the condition \( y_t(x, 0) = 0 \) we choose \( B_n = 0 \) for all \( n \). Thus

\[y(x, t) = (1/10) \cos \pi t \sin 2\pi x.
\]

8. Here \( a = 2 \) and \( L = \pi \). To satisfy the condition \( y(x, 0) = \sin x \) we choose \( A_1 = 1 \) and \( A_n = 0 \) for \( n > 1 \), so

\[y(x, t) = \cos 2t \sin x + \sum B_n \sin 2nt \sin nx,
\]

\[y_t(x, t) = -2 \sin 2t \sin x + \sum 2nB_n \cos 2nt \sin nx.
\]

The condition \( y(x, 0) = 1 \) will be satisfied if \( 2nB_n = 4/\pi n \) for \( n \) odd and \( b_n = 0 \) for \( n \) even. We therefore choose \( B_n = 2/\pi n^2 \) for \( n \) odd and \( B_n = 0 \) for \( n \) even.