4. The Fourier cosine series of the period 4 function $F(t)$ is

$$F(t) = a_0 + \sum a_n \cos(n\pi t/2)$$

where $a_0 = 2$, $a_n = 0$ for $n$ even ($n > 0$), and $a_n = -16/n^2\pi^2$ for $n$ odd. When we substitute this series and the steady periodic trial solution

$$x(t) = A_0 + \sum A_n \cos(n\pi t/2)$$

in the differential equation $x'' + 4x = F(t)$, we find that $A_0 = a_0/4 = 1/2$ and

$$(4 - n^2\pi^2/4)A_n = a_n$$

for $n > 0$. Therefore we choose

$$A_n = 64/n^2\pi^2(n^2\pi^2 - 16)$$

for $n$ odd, and $A_n = 0$ for $n$ even.

6. The period $2\pi$ Fourier cosine series of $F(t) = \sin t$ is

$$F(t) = a_0 + \sum a_n \cos nt$$

where $a_0 = 4/\pi$, $a_n = 0$ for $n$ odd, and $a_n = -4/\pi(n^2 - 1)$ for $n$ even ($n > 0$). Substitution of this series and the steady periodic trial solution

$$x(t) = A_0 + \sum A_n \cos nt$$