Review Problems for Exam 3

Problem 1 Evaluate $\iiint_S z\,dxdy$, where $S$ is the portion of the plane $2x + 2y + z = 6$ in the first octant and $\vec{n}$ is the unit outward normal to $S$. 
Problem 2 Evaluate $\int_C y^3 dx + x^3 dy - z^3 dz$ by Stokes’s theorem, where $C : x = 2\cos t, y = 2\sin t, z = \cos t, 0 \leq t \leq 2\pi$, oriented with increasing $t$. 
Problem 3 Let $S$ is the sphere $x^2 + y^2 + z^2 = 1$, $\vec{n}$ is oriented outward. Evaluate the following surface integrals by the divergence theorem.

a) $\iint_S \vec{v} \cdot \vec{n} \, d\sigma$, $\vec{v} = (x^3, y^3, z^3)$.

b) $\iint_S \frac{\partial f}{\partial n} \, d\sigma$, with $f$ harmonic.
Problem 4 Consider the surface integral \( \iint_S \left( 3dydz + dzdx + 2dxdy \right) \),
where \( S \) is the lower-hemisphere \( x^2 + y^2 + z^2 = 1, \ z < 0 \), and the normal \( \vec{n} \) points downward.
a) Evaluate the above surface integral directly by using following expression for \( S \):
\( S : z = -\sqrt{1 - x^2 - y^2} \)
b) Evaluate the above surface integral by using following parametric expression

\[ S : x = \sin u \cos v, \ y = \sin u \sin v, \ z = \cos u, \ \frac{\pi}{2} \leq u \leq \pi, \ 0 \leq v \leq 2\pi. \]
c) (Challenging) Evaluate the above surface integral by the divergence theorem.
Problem 5 Suppose $\mathbf{F}$ is irrotational and solenoidal. Show that we can write $\mathbf{F} = \nabla f$, where $\nabla^2 f = 0$. 