Review Problems for Exam 2

**Problem 1** Let \( \vec{F} = y \cos x \hat{i} + x \sin y \hat{j} \).

a) Calculate \( \text{div} \vec{F} \) and \( \text{curl} \vec{F} \).

b) Show that \( \vec{F} \) is not a gradient vector field.

**Problem 2** Evaluate \( \iint_{R_{xy}} \exp \left( \frac{y/x}{x/y} \right) dxdy \), where \( R_{xy} \) is the triangular region with vertices \( (0, 0), (0, 1) \), and \( (1, 0) \) by using the change of variables.
Problem 3 Find the area of surface with the parametric equation $x = (2 + \cos v) \cos u,$
$y = (2 + \cos v) \sin u,$ $z = \sin v,$ where $0 \leq u < 2\pi,$ $0 \leq v < 2\pi.$
**Problem 4** Evaluate $\int_{0}^{1} x^2 \sin x^2 \, dx$ by differentiating $\int_{0}^{1} x \sin ax^2 \, dx$ with respect to $a$ twice. (hint: $\int_{0}^{1} x \sin ax^2 \, dx = \frac{1}{2a} - \frac{1}{2a} \cos a$.)
Problem 5 Evaluate \( \int_{C}^{(1,2)} (x^2 - y)dx + (y^2 + x)dy \), where \( C \) is the parabola \( y = x^2 + 1 \).

Problem 6 Evaluate \( \int_{C} (x^2 + y^2)ds \), where \( C \) is the circle \( x^2 + y^2 = 4 \), oriented counterclockwise.
Problem 7 Let $F(x,y) = x^2 - y^2$. Evaluate $\int_{C, (0,0)}^{(2,8)} \nabla F \cdot d\mathbf{r}$, where $C$ is the curve $y = x^3$.

Problem 8 Evaluate the following integrals.

a) $\int_{C, (0,0)}^{(1,1)} y \cos x\,dx + \sin x\,dy$, where $C$ is given by $x = t$, $y = t^3$.

b) $\int_{C} y \cos x\,dx + \sin x\,dy$, where $C$ is the square $|x| + |y| = 1$. 
c) \[ \int_c \frac{x^2dx - y^3dy}{(x^2+y^2)^2} \] around the square with vertices \((\pm 1, \pm 1)\).
d) \[ \int_C \frac{x^2yz - x^3 dy}{(x^2 + y^2)^2}, \text{ where } C \text{ is the circle } (x - 2)^2 + y^2 = 1. \]