Problem 1 (6 pt) Let \( f(x) = \frac{5 - 3x}{2} \) and \( g(x) = \frac{5 - 2x}{3} \). Verify that \( f(x) \) and \( g(x) \) are inverse function of each other by using composite of functions.

solution)

\[
\begin{align*}
    f(g(x)) &= f\left(\frac{5 - 3x}{3}\right) = \frac{5 - (5 - 2x)}{2} = x. \\
g(f(x)) &= g\left(\frac{5 - 3x}{2}\right) = \frac{5 - (5 - 3x)}{3} = x.
\end{align*}
\]

Problem 2 (5 pt each) Find the inverse function if it exists and explain your answer if it does not exist.

a. \( y = 2x^3 - 1 \).

solution) It is 1-1 function. So, there is an inverse function. Switch \( x \) and \( y \). Then,

\[
x = 2y^3 - 1.
\]

\[
2y^3 = x + 1.
\]

\[
y^3 = \frac{x + 1}{2}.
\]

\[
y = \sqrt[3]{\frac{x + 1}{2}}.
\]

Hence the inverse function is

\[
y = \sqrt[3]{\frac{x + 1}{2}}.
\]

b. \( y = x^2 + 2 \).

solution) It is note 1-1 function. So, there is no inverse function.

Problem 3 (5 pt each) Evaluate the followings.

a. \( \log_{27} 9 \)

solution) Note that both 9 and 27 are powers of 3. Using the change of base formula, we get

\[
\log_{27} 9 = \frac{\log_3(3)^2}{\log_3(3)^3} = \frac{2\log_33}{3\log_33} = \frac{2}{3}.
\]

b. \( \ln \sqrt{e} \)
\[\ln \sqrt{e} = \ln e^{\frac{1}{2}} = \frac{1}{2} \ln e = \frac{1}{2}.\]

**Problem 4** (8 pt) Sketch the graphs of \(y = 2^x\), \(y = 3^x\), and \(y = 2^{-x}\). **Indicate the y-intercept.**

solution) If \(x = 0\), \(2^0 = 3^0 = 2^{-0} = 1.\)
So, y-intercept is \((0, 1)\).
The graph which is closer to y axis is \(3^x\) and the graph which is decreasing is \(2^{-x}\).

**Problem 5** (5 pt each) Let \(y = \log(x - 2)\).

a. Sketch the graph.

solution)

b. Find the x-intercept.
0 = \ln(x - 2).

x - 2 = 1.

x = 3.

c. Find the domain.

solution)

x - 2 > 0.

x > 2.

Problem 6 (5+8+7+9 pt) Solve the following equations.

a. $5^x = 4$

solution)

$x = \log_5 4.$

b. $e^{2x} - 3e^x + 2 = 0$

solution) Let $u = e^x$. Then,

$u^2 - 3u + 2 = 0.$

$(u - 1)(u - 2) = 0.$

$u = 1, \quad u = 2.$

$e^x = 1, \quad e^x = 2.$

$x = \ln 1, \quad \ln 2.$

$x = 0, \quad \ln 2.$

c. $2 \log_9(2x) + 1 = 0$

solution)

$2 \log_9(2x) = -1.$

$\log_9(2x) = -\frac{1}{2}.$

$2x = 9^{-\frac{1}{2}}.$
\[ x = \frac{9^{-\frac{1}{2}}}{2}. \]

\[ x = \frac{(3^2)^{-\frac{1}{2}}}{2} = \frac{3^{-1}}{2} = \frac{1}{2 \cdot 3} = \frac{1}{6}. \]

d. \( \log_3 x + \log_3 (x - 1) = \log_3 12 \)

\[ \log_3 x(x - 1) = \log_3 12. \]

\[ x(x - 1) = 12. \]

\[ x^2 - x - 12 = 0. \]

\[ (x - 4)(x + 3) = 0. \]

\[ x = 4, \quad -3. \]

Check: \( x = -3 \): \( \log_3 (-3) \) is not defined. So, \( x = -3 \) is not a solution.

\( x = 4 \): \( \log_3 4, \log_3 (4 - 1) \) are defined and \( \log_3 4 + \log_3 3 = \log_3 12 \). Hence, the solution is \( x = 4 \).