MATH 116 Practice Exam 3  
Key

Problem 1 Let $y = -3x^2 + 6x + 2$. Write it in the standard form and sketch the graph.

solution)

\[ y - 3 = -3(x^2 - 2x + 1) + 2. \]

\[ y = -3(x^2 - 2x + 1) + 5. \]

\[ y = -3(x - 1)^2 + 5. \]

Or,

\[ \frac{y}{-3} = x^2 - 2x - \frac{2}{3}. \]

\[ \frac{y}{-3} + 1 = x^2 - 2x + 1 - \frac{2}{3}. \]

\[ \frac{y}{-3} + 1 = (x - 1)^2 - \frac{2}{3}. \]

\[ y - 3 = -3(x - 1)^2 + 2. \]

\[ y = -3(x - 1)^2 + 5. \]

Problem 2 Solve the equation $6x^3 + 19x^2 + 2x - 3 = 0$. 
solution) Possible rational solutions are
\[ \frac{\pm 1, \pm 3}{\pm 1, \pm 2, \pm 3, \pm 6} = \pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}. \]

By trial and error, we can find a solution \( x = -3 \). Now, using synthetic division, we get
\[(x + 3)(6x^2 + x - 1) = (x + 3)(2x + 1)(3x - 1) = 0.\]
\[x = -3, -\frac{1}{2}, \frac{1}{3}.\]

**Problem 3** Let \( f(x) = 5(x + 1)(x - 2)^2 \). Sketch the graph and indicate the \( y \)-intercept.

solution) \( y \)-intercept is \( y = 5(0 + 1)(0 - 2)^2 = 20 \). Note that the \( x \)-intercepts are \( x = -1, 2 \).

![Graph of the function](image)

**Problem 4** Let \( f(x) = \frac{x^2 - x - 2}{x + 2} \).

a. Find the vertical asymptote.
solution) \( x + 2 = 0, \quad x = -2. \)

b. Find the horizontal/slant asymptote if there is any.
solution) Since the degree of numerator is bigger than the degree of denominator, we can find the slant asymptote. By using long division, we find that the quotient is \( x - 3 \). Hence, the slant asymptote
\[ y = x - 3. \]

c. Find \( x \)-intercept and \( y \)-intercept.
solution) \( x \)-intercept: \( x^2 - x - 2 = 0, \quad (x - 2)(x + 1) = 0, \quad x = -1, 2. \)
y-intercept: $y = -1$.

d. Sketch the graph.

solution) The line here represents the slant asymptote.

<table>
<thead>
<tr>
<th>$x \to -2$</th>
<th>test point</th>
<th>sign of $f(x)$</th>
<th>behavior of $f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x \to -2 -$</td>
<td>$-3$</td>
<td>$\frac{9+3-2}{-3+2}$, so $-$</td>
<td>$-\infty$</td>
</tr>
<tr>
<td>$x \to -2 +$</td>
<td>$-\frac{3}{2}$</td>
<td>$\frac{\frac{9}{2}+\frac{3}{2}-2}{-\frac{3}{2}+2}$, so $+$</td>
<td>$+\infty$</td>
</tr>
</tbody>
</table>

Problem 5 Let $f(x) = x^5 + 1$ and $g(x) = \sqrt[3]{x - 1}$. Verify that they are inverse functions of each other by using composite of functions.

solution)

\[ f(g(x)) = f(\sqrt[3]{x - 1}) = (\sqrt[3]{x - 1})^5 + 1 = x - 1 + 1 = x. \]

\[ g(f(x)) = g(x^5 + 1) = \sqrt[3]{x^5 + 1 - 1} = \sqrt[3]{x^5} = x. \]

Problem 6 Let $y = \frac{5}{3} - \frac{2x}{3}$. Find its inverse function.

solution) Switch $x$ and $y$. Then,

\[ x = \frac{5}{3} - \frac{2y}{3}. \]

\[ 3x = 5 - 2y. \]

\[ 3x - 5 = -2y. \]

\[ y = -\frac{3x}{2} + \frac{5}{2}. \]
**Problem 7** Sketch the graph of \( y = e^{2x} \) and \( y = e^{-2x} \). Indicate the \(-y\)-intercept.

**Solution** \(-y\)-intercept is \( y = e^0 = e^{-0} = 1 \).

Note that \( e^{2x} \) is increasing and \( e^{-2x} \) is decreasing.

**Problem 8** Sketch the graph of \( y = \ln(x - 5) \) and find the domain.

**Solution** Domain: \( x - 5 > 0 \), \( x > 5 \).

**Problem 9** A sample of 500 grams of radioactive lead 210 decay to polonium 210 according to the function defined by \( A(t) = 500e^{-0.032t} \) where \( t \) is time in years. Find the half-life. Leave your answer as an expression. (Half-life: the amount of time which it takes for a quantity to become half of its initial amount).

**Solution**

\[ 250 = 500e^{-0.032t}. \]
\[
\frac{1}{2} = e^{-0.032t}.
\]

\[
\ln \frac{1}{2} = -0.032t.
\]

\[
t = \frac{\ln \frac{1}{2}}{-0.032} = \frac{\ln 2}{0.032} = \frac{1000 \ln 2}{32} = \frac{125 \ln 2}{4}.
\]

**Problem 10** Solve the following equations.

a. \(e^x = -10\)

solution) Since exponential function is always positive, there is no solution. Or,

\[x = \ln(-10).\]

But \(\ln(-10)\) is not defined. Thus, there is no solution.

b. \(\log x - \log(x - 1) = 1\)

solution)

\[\log \frac{x}{x-1} = 1.\]

\[
\frac{x}{x-1} = 10.
\]

\[
x = 10(x - 1).
\]

\[
x = 10x - 10.
\]

\[
9x = 10.
\]

\[
x = \frac{10}{9}.
\]

c. \(2 \ln x + \ln 5 = \ln 20\).

solution)

\[\ln 5x^2 = \ln 20.\]

\[5x^2 = 20.\]

\[x^2 = 4.\]

\[x = 2, -2.\]

Check: \(x = -2\): \(\ln(-2)\) is not defined. So, \(x = -2\) is not a solution.
\[ x = 2: \quad 2 \ln 2 + \ln 5 = \ln 20. \]
The solution is \( x = 2. \)

d. \( 3^{x+2} = \left(\frac{1}{27}\right)^{2x-1}. \)

solution) Note that 27 is a powers of 3, that is, \( 27 = 3^3. \) So,
\[ \frac{1}{27} = \frac{1}{3^3} = 3^{-3}. \]

Hence, we have
\[ 3^{x+2} = (3^{-3})^{2x-1}. \]
\[ 3^{x+2} = 3^{-6x+3}. \]
\[ x + 2 = -6x + 3. \]
\[ 7x = 1. \]
\[ x = \frac{1}{7}. \]