MATH 116 EXAM 3

You need to show all your work to get a full credit.

**Problem 1** (15 points) Let \( y = 3x^2 + 18x + 25 \). Write the function in the standard form and sketch the graph.

**Solution**

\[
y + 27 = 3(x^2 + 6x + 9) + 25.
\]

\[
y = 3(x + 3)^2 - 2.
\]

\[
\begin{array}{c|ccc|ccc|ccc}
& & & & & & & & & \\
\hline
-10 & -7.5 & -5 & 2.5 & 0 & 2.5 & 5 & & & \\
30 & 25 & 20 & 15 & 10 & 5 & 0 & & & \\
\end{array}
\]

**Problem 2** (7 points) The equation \( 2x^3 + 11x^2 + 18x + 9 = 0 \) has a solution \( x = -3 \). Using synthetic division, find all the solutions to the equation.

**Solution**

Using synthetic division, we have

\[
2x^3 + 11x^2 + 18x + 9 = (x + 3)(2x^2 + 5x + 3) = (x + 3)(2x + 3)(x + 1) = 0.
\]

\[
x = -3, \quad -\frac{2}{3}, \quad 1.
\]

**Problem 3** (5 point each) Let \( y = \frac{x^2 - 2x - 15}{x + 1} \).

a. Find the vertical asymptote.

**Solution**

\[
x + 1 = 0.
\]

\[
x = -1.
\]
b. Find the slant asymptote/horizontal asymptote if there is.
   solution) Since the degree of the numerator is bigger than the degree of the
   numerator, there is a slant asymptote. If you divide $x^2 - 2x - 15$ by $x + 1$, the quotient is
   $x - 3$. Hence, the slant asymptote is
   
   $$y = x - 3.$$  

   c. Find the $x$-intercept and $y$-intercept.
   solution) $x$-intercept is

   $$0 = x^2 - 2x - 15.$$  

   $$(x - 5)(x + 3) = 0.$$  

   $$x = 5, \quad -3.$$  

   $y$-intercept is

   $$y = \frac{-15}{1} = -15.$$  

   d. Sketch the graph.
   solution)  

   ![Graph](image)

   **Problem 4** (6 points) Find the inverse function of $y = x^7 + 1$.
   solution) Switch $x$ and $y$. Then,

   $$x = y^7 + 1.$$  

   So,
\[ y^7 = x - 1. \]

\[ y = \sqrt[3]{x - 1}. \]

**Problem 5** (11 points) Sketch the graph of \( y = e^x \) and \( y = e^{-x} \). Indicate the \( y \)-intercepts. Also find the domain and the range.

**Solution** \( y \)-intercept is

\[ y = e^0 = e^{-0} = 1. \]

Domain is \((-\infty, \infty)\) and the range is \((0, \infty)\).

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**Problem 6** (5 point each) Let \( y = \ln(x - 3) \).

a. Find the \( x \)-intercept.

**Solution**

\[ 0 = \ln(x - 3). \]

\[ x - 3 = 1. \]

\[ x = 4. \]

b. Sketch the graph.

**Solution**
c. Find the domain. 

solution) 

\[ x - 3 > 0. \]

\[ x > 3. \]

**Problem 7** (9+9+7 points) Solve the following equations. 

a. \( \log_5 x - \log_5(x - 4) = 1 \) 

solution) 

\[ \log_5 \frac{x}{x - 4} = 1. \]

\[ \frac{x}{x - 4} = 5. \]

\[ x = 5(x - 4). \]

\[ x = 5x - 20. \]

\[ 4x = 20. \]

\[ x = 5. \]

b. \( \ln(x - 2) + \ln(x + 3) = \ln 14. \) 

solution)
\[ \ln(x - 2)(x + 3) = \ln 14. \]

\[
(x - 2)(x + 3) = 14. 
\]

\[ x^2 + x - 6 = 14. \]

\[ x^2 + x - 20 = 0. \]

\[ (x + 5)(x - 4) = 0. \]

\[ x = -5, \quad 4. \]

Check: \( x = -5 \): \( \ln(-7) \) is not defined. Hence it is not a solution.

\( x = 4 \): \( \ln 2 + \ln 7 = \ln 14. \)

Thus, the solution is \( x = 4 \).

\[ c. \left( \frac{2}{3} \right)^{x-2} = \left( \frac{27}{8} \right)^{2x-1} \]

Solution)

\[ \left( \frac{2}{3} \right)^{x-2} = \left( \frac{2}{3} \right)^{-3(2x-1)} \]

\[ x - 2 = -3(2x - 1). \]

\[ x - 2 = -6x + 3. \]

\[ 7x = 5. \]

\[ x = \frac{5}{7}. \]

**Problem 8 (1 point) Print your name:**