Problem 5.  (Bredon Exercise I.3.1) (Munkres Exercise 2.17.6) Let $X$ be a topological space.

a. Let $A$ and $B$ be subsets of $X$. Show the equality $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

b. Let $\{A_\alpha\}$ be a family of subsets of $X$. Show the inclusion $\bigcup_\alpha \overline{A_\alpha} \subseteq \overline{\bigcup_\alpha A_\alpha}$.

c. Find an example where the inclusion in part (b) is strict, and $X$ is a metric space.

Problem 6.  Let $X$ be a metric space and $A \subseteq X$ a subset. The distance from a point $x \in X$ to the subset $A$ is

$$d(x, A) := \inf_{a \in A} d(x, a).$$

Show the equivalence $x \in \overline{A}$ if and only if $d(x, A) = 0$.

Problem 7.  (Munkres Exercise 2.17.13) The diagonal of a space $X$ is the set

$$\Delta := \{(x, x) \mid x \in X\} \subseteq X \times X.$$ 

Show that $X$ is Hausdorff if and only if the diagonal $\Delta$ is closed in $X \times X$. 