**Problem 1.** Let \( X \) be a compact topological space, and \((Y,d)\) a metric space. Consider the uniform metric
\[
d(f, g) := \sup_{x \in X} d(f(x), g(x))
\]
on the set of continuous maps \( C(X, Y) \).
Show that the topology on \( C(X, Y) \) induced by the uniform metric is the compact-open topology.

**Problem 2.** Let \( X \) and \( Y \) be topological spaces. Let \( f, g: X \to Y \) be two continuous maps.
Show that a homotopy from \( f \) to \( g \) induces a (continuous) path from \( f \) to \( g \) in the space of continuous maps \( C(X, Y) \) endowed with the compact-open topology.

More precisely, let \( F(X, Y) \) denote the set of all functions from \( X \) to \( Y \). There is a natural bijection of sets:
\[
\varphi: F(X \times [0, 1], Y) \cong F([0, 1], F(X, Y))
\]
sending a function \( H: X \times [0, 1] \to Y \) to the function \( \varphi(H): [0, 1] \to F(X, Y) \) defined by
\[
\varphi(H)(t) = H(\cdot, t) = h_t.
\]
Your task is to show that if a function \( H: X \times [0, 1] \to Y \) is continuous, then the following two conditions hold:

1. \( h_t: X \to Y \) is continuous for all \( t \in [0, 1] \);
2. The corresponding function \( \varphi(H): [0, 1] \to C(X, Y) \) is continuous.

**Remark.** If \( X \) is locally compact Hausdorff, then the converse holds as well: the two conditions guarantee that \( H: X \times [0, 1] \to Y \) is continuous. In that case, a homotopy from \( f \) to \( g \) is really the same as a path from \( f \) to \( g \) in the function space \( C(X, Y) \).