**Definition.** Let $X$ be a topological space. A function $f: X \to \mathbb{R}$ is **lower semicontinuous** if for all $a \in \mathbb{R}$, the preimage $f^{-1}(a, +\infty)$ is open in $X$.

Equivalently: For all $x_0 \in X$ and $\epsilon > 0$, there is a neighborhood $U$ of $x_0$ satisfying $f(x) > f(x_0) - \epsilon$ for all $x \in U$. This means that the values close to $x_0$ can “suddenly jump up” but not down.

**Problem 2 again.**

b. Let $X$ be a Baire space and $f: X \to \mathbb{R}$ a lower semicontinuous function. Show that for every non-empty open subset $U \subseteq X$, there is a non-empty open subset $V \subseteq U$ on which $f$ is bounded above.

**Problem 3.** Show that a topological space $X$ is of second category in itself if and only if any countable intersection of open dense subsets of $X$ is non-empty.

**Problem 4.** (Uniform boundedness principle) (Willard Exercise 25D.5) (Munkres Exercise 48.10) (Bredon I.17.2)

Let $X$ be a Baire space and $S \subseteq C(X, \mathbb{R})$ a collection of real-valued continuous functions on $X$ which is pointwise bounded: for each $x \in X$, there is a bound $M_x \in \mathbb{R}$ satisfying

$$|f(x)| \leq M_x \text{ for all } f \in S.$$ 

Show that there is a non-empty open subset $U \subseteq X$ on which the collection $S$ is uniformly bounded: there is a bound $M \in \mathbb{R}$ satisfying

$$|f(x)| \leq M \text{ for all } x \in U \text{ and all } f \in S.$$