
Name: ____________________________  Section Code: ____________

Three points will be deducted if these instructions are not followed.

1. Write your full name legibly above.
2. Write your section code from the list in the boxes above.
3. Code your name, netid, and test form F correctly on the scantron form.

CDC – WF 10:00-10:50 -Instructor: Menezes, Glen
CDD – WF 11:00-11:50 -Instructor: Menezes, Glen
CDE – WF 12:00-12:50 -Instructor: Mastroeni, Matthew
CDF – WF 1:00-1:50 -Instructor: Butler, Stacey
CDG – WF 2:00-2:50 -Instructor: Butler, Stacey
CDH – WF 3:00-3:50 -Instructor: Mastroeni, Matthew
CDA – WF 8:00-8:50 -Instructor: Orlow, Nathan
CDJ – WF 11:00-11:50 -Instructor: Orlow, Nathan
DDG – WF 2:00-2:50 -Instructor: Jang, Donghoon
DDH – WF 3:00-3:50 -Instructor: Golze, Hiram
DDA – WF 8:00-8:50 -Instructor: Ahmed, Iftikhar
DDF – WF 1:00-1:50 -Instructor: Vellis, Vyron
DDD – WF 11:00-11:50 -Instructor: Heersink, Byron
DDC – WF 10:00-10:50 -Instructor: Heersink, Byron
DDE – WF 12:00-12:50 -Instructor: Golze, Hiram

• Answers must be marked on scantron form.
• You must not communicate with other students during this test.
• No written materials of any kind allowed.
• The only scratch paper allowed is the last page of the exam. It may be detached.
• No phones, calculators, iPods or electronic devices of any kind are allowed for ANY reason, including checking the time (you may use a simple wristwatch).
• Do not turn this page until instructed to.
• There are several different versions of this exam.

Violations of academic integrity (in other words, cheating) will be taken extremely seriously, and will be handled under the procedures of Article I, Part 4 of the student code.
Mark answers on scantron form.

Your test form is H. Code this on the scantron form now.

Choose the correct answer for each given sequence \( \{a_n\} \).

1. \( a_n = n3^{-n} \)
   
   (A) Converges to 1
   (B) Converges to 3
   (C) Diverges
   (D) Converges to \( 3^{-1} \)
   (E) Converges to 0

2. \( a_n = \frac{2n^4 + 5n^3 + 2}{n^4 + 2n^3} \).
   
   (A) Converges to 1
   (B) Converges to 2
   (C) Converges to \( \frac{5}{2} \)
   (D) Converges to \( \frac{5}{4} \)
   (E) Diverges

3. \( a_n = \frac{\cos(n)}{2\sqrt{n^2} + 1} \).
   
   (A) Converges to 1
   (B) Converges to \( \frac{1}{\sqrt{2}} \)
   (C) Converges to 0
   (D) Converges to \( \frac{1}{2} \)
   (E) Diverges
For each series mark \textbf{C} if the series \textbf{Converges} or \textbf{D} if the series \textbf{Diverges}. Mark answers on Scantron form.

4. \[ \sum_{n=0}^{\infty} \frac{n^3 + 7n}{5^n + n^2} \]

5. \[ \sum_{n=1}^{\infty} \frac{n^{\frac{1}{2}}}{13 + n^{\frac{3}{2}}} \]

6. \[ \sum_{n=1}^{\infty} \frac{n^3 + n^6 + 15n}{n^7 + n^3} \]

7. \[ \sum_{n=1}^{\infty} \left( \frac{\pi}{4} \right)^n \]

8. \[ \sum_{n=2}^{\infty} \sin \left( \frac{1}{n^2} \right) \]

9. \[ \sum_{n=1}^{\infty} \left( \frac{\pi}{3} \right)^n \]
10. Complete the statement of the monotone convergence theorem: if a sequence \( \{a_n\} \) is \underline{____________} then it converges.

(A) increasing and bounded
(B) increasing and positive
(C) positive and bounded
(D) positive and continuous
(E) continuous and bounded

11. The velocity of a runner during the first 6 seconds of running is shown. Use the midpoint rule with three intervals to estimate the distance traveled by the runner.

\[
\begin{array}{c|c}
\text{t (s)} & v (\text{m/s}) \\
0 & 0 \\
1 & 2 \\
2 & 3 \\
3 & 3 \\
4 & 4 \\
5 & 5 \\
6 & 6 \\
\end{array}
\]

(A) 30 m
(B) 10 m
(C) 12 m
(D) 24 m
(E) 20 m

12. Find the sum of the series \( \sum_{n=1}^{\infty} \frac{e^n}{3^{2n-1}} \).

(A) \( \frac{e}{9-e} \)
(B) \( \frac{e}{3(9-e)} \)
(C) \( \frac{3e}{9-e} \)
(D) Diverges
(E) \( \frac{9e}{9-e} \)
For the problems 13, 14 and 15, consider the curve $y = \sqrt{b^2 - x^2}$ between the points $0 \leq x \leq b$.

13. Which of the following is an integral which gives the length of the curve?

(A) $\int_{0}^{b} \frac{b}{\sqrt{b^2 - x^2}} dx$

(B) $\int_{0}^{b} \frac{b^2}{b^2 - x^2} dx$

(C) $\int_{0}^{b} \frac{1}{\sqrt{b^2 - x^2}} dx$

(D) $\int_{0}^{b} dx$

(E) $\int_{0}^{b} b dx$

14. Which one is an integral which represents the surface area when the curve is rotated about the $x$-axis?

(A) $2\pi \int_{0}^{b} \frac{b^2}{b^2 - x^2} dx$

(B) $\int_{0}^{b} 2\pi dy$

(C) $2\pi \int_{0}^{b} \frac{1}{\sqrt{b^2 - y^2}} dy$

(D) $2\pi \int_{0}^{b} b dx$

(E) $2\pi \int_{0}^{b} \frac{b}{\sqrt{b^2 - y^2}} dx$

15. What is the surface area?

(A) $2\pi b$

(B) $2\pi b^2$

(C) 1

(D) $\pi b$

(E) $\pi b^2$
16. Test the series \( \sum_{n=1}^{\infty} \frac{(100)^n}{(n + 1)!} \) for convergence.

(A) Absolutely convergent
(B) Divergent
(C) Conditionally convergent

17. Test the series \( \sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^3} + 2} \) for convergence.

(A) Conditionally convergent
(B) Absolutely convergent
(C) Divergent

18. If \( s_n = 1 - \frac{1}{5^n} \), what is \( a_n \) for \( n > 1 \)?

(A) \( 2 + \frac{1}{5n(n-1)} \)
(B) \( 2 \)
(C) \( 2 + \frac{2}{5n(n-1)} \)
(D) \( \frac{1}{5n(n-1)} \)
(E) \( \frac{2}{5n(n-1)} \)

19. If \( s_n = 3 + \frac{2}{3^n} \), what is \( \sum a_n \) for \( n \geq 1 \)?

(A) 0
(B) \( \frac{1}{3} \)
(C) \( \frac{2}{3} \)
(D) \( \frac{3}{2} \)
(E) \( \sqrt{3} \)

20. What is the smallest value of \( n \) which guarantees that the partial sum \( S_n \) approximates the series \( \sum_{k=1}^{\infty} \frac{1}{k^3} \) to within \( \frac{1}{800} \)?

(A) 20
(B) 22
(C) 11
(D) 10
(E) 19
21. Find the sum of the series \( \sum_{n=1}^{\infty} \left( \frac{1}{\sqrt{n+1}} - \frac{1}{\sqrt{n+3}} \right) \) if it is convergent.

(A) Divergent  
(B) \( \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} \)  
(C) \( \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} \)  
(D) \( \frac{1}{\sqrt{3}} \)  
(E) \( \frac{1}{\sqrt{2}} \)

22. Consider an infinite array of circles arranged in rows as shown. In row one there is a circle of radius 1. In row two there are two circles of radius 1/3. In row three there are 2 circles of radius 1/9. In row four there are two circles of radius 1/27, and so on, forever. What is the total area enclosed by all of the circles?

\[ \text{Area} = \sum_{n=1}^{\infty} \frac{\pi}{3^{n-1}} \]

(A) \( \frac{\pi}{6} \)  
(B) \( \frac{\pi}{4} \)  
(C) \( \frac{5\pi}{4} \)  
(D) \( \frac{5\pi}{9} \)  
(E) \( \frac{5\pi}{3} \)
23. A vertical door has the shape of the parabola $y = x^2$, $-1 \leq x \leq 1$. The top of the door is 3 meters under water. What is the hydrostatic force on the door?

($\rho$ kg/m$^3$ is the density of water and $g$ m/s$^2$ is the acceleration due to gravity.)

(A) $\rho g \frac{8}{5}$
(B) $\rho g \frac{92}{15}$
(C) $\rho g \frac{68}{15}$
(D) $\rho g \frac{16}{5}$
(E) $\rho g \frac{92}{5}$
Scratch paper–may be detached