**Additional practice with polar curves**

(1) For the given polar curves, make a careful sketch and find the total area enclosed.
   a) \( r = \cos 3\theta \)
   b) \( r^2 = \sin 2\theta \)

(2) Find the area of the region enclosed by one loop of \( r = \sin(5\theta) \).

Remember the formula for \( ds \) in polar coordinates:

\[
ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta
\]

(3) Suppose that \( r = f(\theta), \, 0 \leq \theta \leq \pi \). Set up an integral whose value is the surface area when this curve is rotated about the \( x \)-axis. Set up another integral whose value is the surface area when this curve is rotated about the \( y \)-axis.

(4) Set up an integral whose value is the surface area when the lobe of the flower \( r = \sin 5\theta \) from \( 2\pi/5 \) to \( 3\pi/5 \) is rotated about the \( x \)-axis.

(5) Find the area which lies inside \( r = 3\sin\theta \) and outside \( r = 2 - \sin\theta \).

**Sample final questions**

These are sample questions taken from old final exams. This is not a comprehensive list of problems—some topics are not covered. This is not a study guide.

(1) Evaluate the integral: \( \int x^3 \ln x \, dx \).

(2) Evaluate the integral: \( \int \frac{x^3}{\sqrt{9 - x^2}} \, dx \).

(3) You are given the series

\[
\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n}{n^2 + 1}.
\]

Justify your answer completely in each part.
   a) Does the series converge?
   b) Does the series converge absolutely?
   c) Does the series converge conditionally?

(4) Find the radius of convergence and interval of convergence of the power series

\[
\sum_{n=0}^{\infty} \frac{(-1)^n \cdot n \cdot x^n}{10^{n+1}}.
\]

(5) Consider the parametric curve

\( x = 2 \cos t, \, y = 3 \sin t, \, 0 \leq t \leq \pi \).

   a) Eliminate the parameter. Then sketch and identify this curve. Indicate the direction of motion as \( t \) increases.
   b) Set up and simplify but do not evaluate an integral which represents the area under the graph.
   c) Set up and simplify but do not evaluate an integral which represents the arclength of this curve.

(6) a) Make a careful sketch of the curve whose polar equation is \( r = 2 \sin(3\theta) \).

Be sure that all important features are illustrated.

b) Find the total area enclosed by this polar curve.

c) Set up and simplify but do not evaluate an integral which represents the total length of this curve.
Multiple Choice. No partial credit.

(7) Evaluate: \( \int \frac{1}{(x-1)(x+2)} \, dx \).

a) \( \frac{1}{2} \ln |x-1| - \frac{1}{2} \ln |x+2| + C \)  

b) \( \frac{1}{3(x-1)^2} - \frac{1}{3(x+2)^2} + C \)  

c) \( \frac{1}{2} (x-1) - \frac{1}{2} (x+2) + C \)  

d) \( \frac{1}{2(x-1)^2} - \frac{1}{2(x+2)^2} + C \)  

e) \( \frac{1}{3} \ln |x-1| - \frac{1}{3} \ln |x+2| + C \)  

f) \( \ln |x-1| + \ln |x+2| + C \).

(8) The improper integral

\[ \int_{2}^{\infty} \frac{1}{x(\ln x)^2} \, dx \]

a) converges to 1  

b) converges to \( \frac{1}{2} \)  

c) converges to 0  

d) converges to \( \frac{1}{\ln(2)} \)  

e) converges to \( \ln(\ln(2)) \)  

f) converges to 2  

g) diverges  

(9) **TWO** of the following integrals represent the area of the surface generated by rotating the graph of \( y = \ln x \), \( e \leq x \leq e^2 \) about the \( x \)-axis. **Circle exactly two answers.**

I) \( S = 2\pi \int_{e}^{e^2} \ln x \sqrt{1+e^{2x}} \, dx \).  

II) \( S = 2\pi \int_{e}^{e^2} x \sqrt{1+1/x^2} \, dx \).  

III) \( S = 2\pi \int_{e}^{e^2} \ln x \sqrt{1+1/x^2} \, dx \).  

IV) \( S = 2\pi \int_{1}^{2} e^y \sqrt{1+e^{2y}} \, dy \).  

V) \( S = 2\pi \int_{1}^{2} y \sqrt{1+e^{2y}} \, dy \).  

VI) \( S = 2\pi \int_{1}^{2} e^x \sqrt{1+1/x^2} \, dx \).  

(10) The sequence \( a_n = \frac{n^2 + n + 1}{10n^3 + n^2 - 1} \)

a) approaches \( \infty \).  

b) converges to 0  

c) converges to \( \frac{1}{10} \)  

d) Converges to 1.  

e) Diverges by the limit comparison test.  

f) Converges by the limit comparison test.  

g) None of the above.  

(11) The sequence \( a_n = \left( \frac{2}{n} \right)^{3/n} \)

a) approaches \( \infty \).  

b) converges to 0.  

c) converges to \( \frac{2}{3} \)  

d) converges to \( 3/2 \)  

e) converges to 2  

f) converges to 1  

g) none of the above  

(12) Find the sum of the series \( \sum_{n=1}^{\infty} \frac{3^{n+1}}{2^{2n}} \).

a) 9  

b) 9/2  

c) 9/4  

d) 18  

e) 27  

f) 27/2  

g) None of the above.
(13) Consider the series \( \sum_{n=1}^{\infty} \frac{7n^2 + n - 1}{n^4 - 2n + 1} \).

Two of the following statements are true. Circle exactly two statements.

a) The series diverges by the ratio test.
b) The series converges by the ratio test.
c) The ratio test is inconclusive.
d) The series diverges by the limit comparison test.
e) The series converges by the limit comparison test.
f) The series converges to zero.
g) The series diverges by the test for divergence
h) The series converges by the alternating series test.

(14) Which of the following infinite series represents the value of the definite integral \( \int_{0}^{1/2} e^{-x^2} \, dx \).

a) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!} \).
b) \( \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n \cdot n \cdot n!} \).
c) \( \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n \cdot n \cdot n!} \).
d) \( \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n \cdot n!} \).
e) \( \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n \cdot n \cdot n!} \).
f) \( \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}(2n+1) \cdot n!} \).
g) None of the above.

(15) Express the given polar equation in Cartesian coordinates:

\( r^2 = \sin 2\theta \).

a) \( (x^2 + y^2)^2 - 2xy = 0 \)
b) \( (x + y)^2 - 2xy = 0 \)
c) \( x^2 + y^2 - xy = 0 \)
d) \( (x^2 + y^2)^2 - 4(x^2 + y^2) = 0 \)
e) \( (x + y)^2 - 4xy = 0 \)
f) \( x^2 + y^2 - 4xy = 0 \).

(16) The parametric equations for a cycloid are given by

\[ x = t - \sin(t), \quad y = 1 - \cos(t). \]

Find the slope of the tangent line at the point where \( t = \pi/4 \).

a) 1  b) \( \sqrt{2} \)
c) \( \frac{1}{\sqrt{2}} \)  d) \( \sqrt{2} - 1 \)
e) \( \frac{1}{\sqrt{2}} - 1 \)  f) \( -\frac{1}{\sqrt{2}} \)
g) \( -\sqrt{2} \).
(17) You are given several polar equations and several descriptions of polar graphs. Find the best
description for each polar graph and write the number on the blank next to the equation.

a) $r = \cos \theta$. ______  
b) $r = 1 + 2 \cos \theta$. _____
c) $r = 2 + \cos \theta$. ______  
d) $r = \sin(4\theta)$. ______  
e) $r = \theta$. ______

(1) Rose with 4 leaves.  
(2) Rose with 8 leaves.  
(3) Circle with center at (1/2, 0).  
(4) Circle with center at (1, 0).  
(5) Circle with center at origin.  
(6) Circle with center at (0, 1/2).  
(7) Spiral.  
(8) Figure eight oriented on horizontal axis.  
(9) Figure eight oriented on vertical axis.  
(10) A figure with two loops; one inside the other.  
(11) Looks like a circle with one side pushed in.