Let \( C: z = z(t), a \leq t \leq b \), \( f(z) \) defined on \( C \).

Image: \( \Gamma: w = f(z(t)) \), \( a \leq t \leq b \).

Let \( \gamma(t) := z(t) \) \( a < t < b \) and \( f \) is analytic at \( z_0 \), \( f'(z_0) \neq 0 \).

\[ w(t) = f(z(t)) \Rightarrow \]
\[ w'(t_0) = f'(z(t_0)) \cdot z'(t_0) \]

\[ \Rightarrow \quad \arg w'(t_0) = \arg f'(z(t_0)) + \arg z'(t_0) \]

Let: \( \varphi_0 \)

\[ \Rightarrow \quad \text{Then: } \gamma_0 = \arg f'(z_0) \]

Angle of rotation.
Let $C_1, C_2$ be two smooth curves passing through $z_0$.

For $T_1, T_2$,

$$
\phi_1 = T_2 + \Theta_1 \\
\phi_2 = T_0 + \Theta_2
$$

Therefore,

$$
\phi_1 - \phi_2 = \Theta_1 - \Theta_2
$$

at the point $u_0 = f(z_0)$.

Let $\alpha$ be the angle from

Because of this angle preserving property, $w = f(z)$ is said to be

**CONFORMAL** at $z_0$. (Also, check in another of $z$)

**Ex:**

$$
\bar{w} = e^2
$$

**Ex:** $w = \bar{z}$ is not conformal.
Let \( f \) be not constant and \( f'(z_0) = 0 \).

Then \( z_0 \) is called a \underline{critical point} of the transformation \( w = f(z) \).

What happens around critical pt? 

\[ w = 1 + z^2. \]

Thus, it is corr. of \( z = z^2 \) and \( w = 1 + z \).

\underline{Critical pt \( \neq w \):} \( z_0 = 0 \).

Take a ray \( \theta = \alpha \) from the pt \( z_0 = 0 \).

\[ \theta = \beta. \]

\[ \theta = \alpha. \]

\[ 1 + z^2 \]

\[ \text{difference} = \beta - \alpha. \]

\[ \text{difference:} \ 2(\beta - \alpha). \]

So, it is \underline{doubled}.

\[ \therefore \text{if it is critical pt, then} \ f^{(n)}(z_0) \neq 0 \]

Angle the two arcs is multiplied by this \( m. \)