Tuesday, September 10  *  Solutions  *  Visualizing quadric surfaces

Elliptic paraboloid: \( z = Ax^2 + By^2 \) \((A, B \text{ have same sign})\)

- If \( A = 0 \) or \( B = 0 \) our surface becomes a parabola extended out parallel to a coordinate axis. If \( A = B = 0 \) our surface becomes the plane \( z = 0 \). Neither of those surfaces are elliptic.
- If \( A \) and \( B \) were both negative the surface would be a downward opening elliptic paraboloid contained entirely beneath the plane \( z = 0 \).

Hyperbolic paraboloid: \( z = Ax^2 + By^2 \) \((A, B \text{ differ in sign})\)

- The horizontal cross section given by \( z = 0 \) is a set of two crossing lines, which is not a hyperbola.
- \( y^2 - x^2 = -(x^2 - y^2) \) so the two surfaces would be mirrors of each other across the plane \( z = 0 \).

Ellipsoid: \( \frac{x^2}{A^2} + \frac{y^2}{B^2} + \frac{z^2}{C^2} = 1 \)

- To be a sphere we'd need \( A^2 = B^2 = C^2 \)
- The sliders cannot go to 0 since \( A, B \) and \( C \) are divisors in the equation.

Double cone: \( z^2 = Ax^2 + By^2 \)

- Setting \( z \) equal to a constant gives the equation for an ellipse, while setting \( x \) or \( y \) equal to a constant gives the equation for a hyperbola.
- If \( A = 0 \) or \( B = 0 \) the equation yields a set of two intersecting planes.
- The cross sections given by \( x = 0 \) or \( y = 0 \) are sets of two intersecting lines.

Hyperboloid of one sheet: \( \frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{z^2}{C^2} = 1 \)

- The sliders don't go to 0 because \( A, B \) and \( C \) are divisors in the equation.
- When \( x = \pm A \) the equation reduces to \( C^2 y^2 = B^2 z^2 \), which describes two intersecting lines.
- There must always be a hole through the hyperboloid, since when \( z = 0 \) our equation is \( \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1 \), which describes a nontrivial ellipse (if \((x, y)\) is in this ellipse, then so is \((-x, -y)\), and \((0, 0)\) does not satisfy this equation).

Hyperboloid of two sheets: \( -\frac{x^2}{A^2} - \frac{y^2}{B^2} + \frac{z^2}{C^2} = 1 \)

- The larger \( A \) and \( B \) get the smaller the terms \(-\frac{x^2}{A^2}\) and \(-\frac{y^2}{B^2}\) get, making the equation closer to one describing two planes.
- There must always be a gap between the two sheets because the equation cannot be satisfied when \( z = 0 \).