1. Consider the points $A = (2, 0, 1)$ and $B = (4, 2, 5)$ in $\mathbb{R}^3$.

(a) Find the point $M$ which is halfway between $A$ and $B$ on the line segment $L$ joining them. (2 pts)

(b) Find the equation for the plane $P$ consisting of all points that are equidistant from $A$ and $B$. (3 pts)

2. Consider the function
   \[ f(x, y) = \begin{cases} 
   \frac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0), \\
   0 & \text{if } (x, y) = (0, 0). 
   \end{cases} \]

(a) Compute the following limit, if it exists. (4 pts)
   \[ \lim_{(x,y) \to (0,0)} f(x, y) \]

(b) Where on $\mathbb{R}^2$ is the function $f$ continuous? (1 pts)

3. Consider the function $f: \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = xy$.

(a) Use Lagrange multipliers to find the global (absolute) max and min of $f$ on the circle $x^2 + y^2 = 2$. (6 pts)

(b) If they exist, find the global min and max of $f$ on $D = \{x^2 + y^2 \leq 2\}$. (2 pts)

(c) For each critical point in the interior of $D$ you found in part (b), classify it as a local min, local max, or saddle. (2 pt)

(d) If they exist, find the global min and max of $f$ on $\mathbb{R}^2$. (2 pts)

4. A function $f: \mathbb{R}^2 \to \mathbb{R}$ takes on the values shown in the table at right.

   (a) Estimate the partials $f_x(1, 1)$ and $f_y(1, 1)$. (2 pts)

   \[
   \begin{array}{c|cccc}
   x & 0.2 & 0.6 & 1.0 & 1.4 \\
   \hline
   y & 1.8 & 3.16 & 3.88 & 4.60 & 5.32 & 6.04 \\
   \end{array}
   \]

   (b) Use your answer in (a) to approximate $f(1.1, 1.2)$. (2 pts)

   \[
   \begin{array}{c|cccc}
   x & 0.6 & 1.72 & 1.96 & 2.20 & 2.44 & 2.68 \\
   \hline
   y & 1.0 & 1.24 & 1.32 & 1.40 & 1.48 & 1.56 \\
   \end{array}
   \]

   (c) Determine the sign of $f_{xy}(1, 1)$: positive negative zero (1 pt)

5. Consider the region $E$ shown at right, which is bounded by the $xy$-plane, the plane $z = y = 0$ and the surface $x^2 + y = 1$. Complete setup, but do not evaluate, a triple integral that computes the volume of $E$. (6 pts)
6. Match the following functions \( \mathbb{R}^2 \to \mathbb{R} \) with their graphs and contour diagrams. Here each contour diagram consists of level sets \( \{ f(x, y) = c_i \} \) drawn for evenly spaced \( c_i \). (9 pts)

(a) \( \sqrt{8 - 2x^2 - y^2} \)  
(b) \( \cos x \)  
(c) \( xy \)
7. Consider the portion $R$ of the cylinder $x^2 + y^2 \leq 2$ which lies in the positive octant and below the plane $z = 1$. Compute the total mass of $R$ when it is composed of material of density $\rho = e^{x^2 + y^2}$. (7 pts)

8. For the curve $C$ in $\mathbb{R}^2$ shown and the vector field $F = (\ln(\sin(x)), \cos(\sin(y)) + x)$ evaluate $\int_C F \cdot d\mathbf{r}$ using the method of your choice. (5 pts)

9. Let $R$ be the region shown at right.

(a) Find a transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ taking $S = [-1, 1] \times [-1, 1]$ to $R$. (4 pts)

(b) Use your change of coordinates to evaluate $\int_R y^2 \, dA$ via an integral over $S$. (6 pts)

Emergency backup transformation: If you can’t do (a), pretend you got the answer $T(u, v) = (uv, u + v)$ and do part (b) anyway.

10. Consider the surface $S$ which is parameterized by $\mathbf{r}(u, v) = (\sqrt{1 + u^2}\cos v, \sqrt{1 + u^2}\sin v, u)$ for $-1 \leq u \leq 1$ and $0 \leq v \leq 2\pi$.

(a) Circle the picture of $S$. (2 pts)

(b) Completely setup, but do not evaluate, an integral that computes the surface area of $S$. (6 pts)
11. For the cone \( S \) at right, give a parameterization \( \mathbf{r}: D \to S \). Explicitly specify the domain \( D \). (5 pts)

12. Consider the region \( R \) in \( \mathbb{R}^3 \) above the surface \( x^2 + y^2 - z = 4 \) and below the \( xy \)-plane. Also consider the vector field \( \mathbf{F} = (0, 0, z) \).

   (a) Circle the picture of \( R \) below. (2 pts)

   (b) Directly calculate the flux of \( \mathbf{F} \) through the entire surface \( \partial R \), with respect to the outward unit normals. (10 pts)

   (c) Use the Divergence Theorem and your answer in (b) to compute the volume of \( R \). (3 pts)

13. Let \( C \) be the curve shown at right, which is the boundary of the portion of the surface \( x + z^2 = 1 \) in the positive octant where additionally \( y \leq 1 \).

   (a) Label the four corners of \( C \) with their \((x,y,z)\)-coordinates. (1 pt)

   (b) For \( \mathbf{F} = (0, xyz, xyz) \), directly compute \( \int_C \mathbf{F} \cdot d\mathbf{r} \). (6 pts)

   (c) Compute \( \text{curl} \mathbf{F} \). (2 pts)

   (d) Use Stokes’ Theorem to compute the flux of \( \text{curl} \mathbf{F} \) through the surface \( S \) where the normals point out from the origin. (3 pts)

   (e) Give two distinct reasons why the vector field \( \mathbf{F} \) is \textit{not} conservative. (2 pts)
Extra Credit 1: Consider the transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ which distorts the plane as shown below:

\[ T \]
\[ \begin{array}{c}
\text{y} \\
\end{array} \]
\[ \begin{array}{c}
\text{x} \\
\end{array} \]

(a) Draw in $T(0,0)$ on the right-hand part of the picture. (1 pt)

(b) Compute the Jacobian matrix of $T$ at $(0,0)$, taking it as given that the entries of the matrix are integers. Hint: Tear off the bottom of this page to form a makeshift ruler. (3 pts)

Extra Credit 2: Consider the torus $T$ shown below where the inner radius is 2 and the outer radius is 4, and hence the radius of tube itself is 1.

1. Compute the volume of $T$ by computing the flux of some vector field $F$. (3 pts)

2. Compute the volume of $T$ via a 3-dimensional change of coordinates where your final integral is over a rectangular box. (2 pts)