In Calc I & II you learn how to study real valued functions

\[ y = f(x) \]

of a single real variable.

1. **Derivative**: rate of change
   - Slope of tangent line to graph
   \[ f'(x) = \frac{df}{dx} = \frac{\Delta f}{\Delta x} \]

2. **Integral**: signed area under graph
   - Average value
   \[ \frac{1}{b-a} \int_{a}^{b} f(x) \, dx \]

3. **Fundamental THRM of Calculus**: relates the 2:
   \[ f(b) - f(a) = \int_{a}^{b} f(x) \, dx \]

Too constrained — can only study functions with a single real input, and single real output.

Eg: Temperature depends on location & time: on Earth, we need to specify long, lat, time — 3 quantities.

* Your location on Earth at any given time is specified by long/lat — 2 quantities.
**N-dimensional Space** (§12.1)

1-DIML SPACE = R-LINE = REAL NUMBERS \( \mathbb{R} \)

\[ \begin{array}{c}
\emptyset \rightarrow \mathbb{R} \\
\emptyset \rightarrow \mathbb{R}^{2} = \text{Cartesian Plane, Real Plane} \\
\emptyset \rightarrow \mathbb{R}^{3} = \text{Triplets of Real #s} \\
\emptyset \rightarrow \mathbb{R}^{n} = \text{N-tuples of Real #s}
\end{array} \]

WE USE 2-D \& 3-D TO BUILD OUR INTUITION, BUT TOOLS ARE MOSTLY APPLICABLE IN N-D.

(* THERE ARE SOME TECHNIQUES SPECIFIC TO 2-D \& 3-D *)
Plan for This Course

Develop Calculus for These More General Functions

1. Derivatives §14 (13)
2. Integrals §15 (13)
3. "FCT" §16

To get there:

1. What sets Calculus apart from Algebra, Trig, Precalc?
A. Limits!

\[ \lim_{x \to a} f(x) = L \] means "as \( x \) approaches \( a \), \( f(x) \) approaches \( L \)

This requires a notion of proximity...distances.

Define given \( P(x_1, \ldots, x_n), Q(y_1, \ldots, y_n) \in \mathbb{R}^n \) (\( \in \mathbb{R}^n \) = "is an element of \( \mathbb{R}^n \))

\[ |PQ| = \text{Distance from } P \text{ to } Q \]

\[ = \sqrt{(x_1-y_1)^2 + (x_2-y_2)^2 + \cdots + (x_n-y_n)^2} \]

\[ n=2 \]

\[ \sqrt{(x-x')^2 + (y-y')^2} \] (Pythagorean Thm)

\[ n=3 \]

\[ \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} \]
Using this, we have: Equation of a Sphere -

**Defn:** Given a point \( C(a, b, c) \) in \( \mathbb{R}^3 \) and \( r > 0 \), the **Sphere of radius** \( r \) with **center** \( C \) is the set of points \( P(x, y, z) \) with distance \( r \) to \( C \):

\[
\forall \ P \in \mathbb{R}^3: \ |PC| = r^2
\]

\[
|PC| = r \iff \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2} = r
\]

\[
\iff (x-a)^2 + (y-b)^2 + (z-c)^2 = r^2
\]

**Eqn of Sphere** (Set of solns \( (x, y, z) \)) - (0 this eqn is a sphere)

We will revisit eqns for other surfaces later...

In 1-variable, derivatives are defined via **limits and arithmetic** (and interaction).

\( \mathbb{R}^n \): "Arithmetic" involves vectors (§12.2) - next time.

**Syllabus**