1. **Definitions:** For the following questions an answer is sufficient: Just state the requested definition, theorem, formulas, etc. Be sure to use correct notation, include any necessary quantifiers in the appropriate order, and connecting words (e.g., “such that”) if necessary. Definitions should be stated in English, i.e., not using logical symbols (∀, ∃, ¬, =, ⇒, etc.).

(a) A function \( f \) from \( \mathbb{R} \) to \( \mathbb{R} \) is **not decreasing** if ...

(b) The **graph** of a function \( f : A \to B \) is defined as the set ...

(c) Given a function \( f : A \to B \), the **image** of a set \( C \subseteq A \) is the set ...

(d) Let \( A = \{1, 2\} \), and \( B = \{2, 3, 4\} \). Write down explicitly (i.e., with all elements explicitly listed) the following sets. Be sure to use correct set-theoretic notation.

(i) \( A - B = \ldots \)

(ii) \( A \times B = \ldots \)

(iii) The power set of \( A \): \( P(A) = \ldots \)

2. Consider the following statement (where the underlying universe is the set \( \mathbb{N} \) of natural numbers): “If \( n \) is odd, then \( n \) is prime.”

(a) Write the **contrapositive** of this statement.

(b) Write the **converse** of this statement.

(c) Rewrite the statement in the form “... is a sufficient condition for...”.

(d) Write the **negation** of this statement without using words of negation. (You can use “composite” as a synonym for “not prime” and “even” for “not odd”.)

3. **Negations:** Negate the following statements without using words of negation. Write your negation first in symbolic form (i.e., using logical symbols ∀, ∃, ⇒, ¬, and, etc.), then as an English sentence (without using logical symbols). (Here \( f \) is assumed to be a function from \( \mathbb{R} \) to \( \mathbb{R} \).)

(a) If \( x \neq 0 \) and \( y \neq 0 \), then \( f(x,y) \neq 0 \).

(b) For every \( \epsilon > 0 \) there exists \( \delta > 0 \) such that \( |f(x)| < \epsilon \) whenever \( |x| < \delta \).

(c) There exists \( M \in \mathbb{R} \) such that for all \( x \in \mathbb{R} \) there exists \( y > x \) such that \( f(y) \leq M \).

(d) To pass this class requires passing the midterm and the final. (Hint: To avoid words of negation, use the word fail as the opposite of “pass”).

4. Let \( f \) be a function from \( \mathbb{R} \) to \( \mathbb{R} \) and let \( g \) be defined by \( g(x) = e^{f(x)} \) for all \( x \in \mathbb{R} \).

(a) Prove that if \( f \) is bounded, then \( g \) is bounded. (You may use the standard properties of the exponential function.)

(b) Give an example showing that converse is not true. (No proof is required for this part. Just give a counterexample.

5. Let \( f : A \to B \) be a function, and let \( C \) and \( D \) be subsets of \( A \).

(a) Give a careful, step-by-step proof of the relation \( f(C \cap D) \subseteq f(C) \cap f(D) \).

    Your write-up must include all necessary steps, with appropriate justifications (e.g., “by the def. of ...”), in the correct logical order, use proper notation and terminology, and include any necessary quantifiers and connecting words (e.g., “therefore”, “such that”).

(b) Give an example showing that equality does not hold. (No formal proof required for this part. A brief explanation is enough.)