Solve the following problems and hand in your solutions.


(7) Prove that
\[
\lim_{n \to \infty} \frac{2^n \cdot n!}{n^n} = 0.
\]

Hint: Look for what Stirling’s Approximation is.

(8) Prove that
\[
\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1.
\]

(9) Let \( u_n = \sqrt{u_n + 1} \) and \( u_1 = 1 \). Prove that
\[
\lim_{n \to \infty} u_n = \frac{1}{2} (1 + \sqrt{5})
\]

(10) Evaluate
\[
\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{5^n}
\]