1. (25%) The current price of an annual coupon bond is 100. The derivative of the price of the bond with respect to the yield to maturity is -700. The yield to maturity is an annual effective rate of 8%.

Calculate the duration of the bond.

\[
\frac{dP}{di} = 7.56
\]

With \( \frac{dP}{di} \), calculate Modified Duration first

\[
DM = \frac{\frac{dP}{di}}{P} = \frac{-700}{100} = 7
\]

by def. \( \frac{D}{1+i} \)

\[
D = 7 \times 1.08 = 7.56
\]

2. (25%) Calculate the duration of a common stock that pays dividends at the end of each year into perpetuity. Assume that the dividend is constant, and that the effective rate of interest is 10%.

\[
D = \frac{\sum_{t=1}^{\infty} \frac{1}{t} \cdot \frac{1}{(1+i)^t}}{\sum_{t=1}^{\infty} \frac{1}{(1+i)^t}} = \frac{1}{i} + \frac{1}{i^2} = \frac{1+i}{i} = 11
\]

To prove the numerator.

Let \( S = 1 + \frac{1}{i} + \frac{1}{i^2} + \ldots \) then \( v \cdot S = v + \frac{v}{i} + \frac{v}{i^2} + \ldots \) Substract \( v \cdot S \) from \( S \).

\[
(1-v) \cdot S = v + v^2 + v^3 + \ldots = \frac{1}{1-v}
\]

\[
S = \frac{1}{1+i}
\]

Your answers: (Leave blank if you need no grading)

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3. (25%) Calculate the duration of a common stock that pays dividends at the end of each year into perpetuity. Assume that the dividend increases by 2% each year and that the effective rate of interest is 5%.

\[
D = \frac{\sum_{t=1}^{\infty} \frac{t \cdot 0.02}{1.02} t^{-1}}{\sum_{t=1}^{\infty} \frac{1}{1.02} t^{-1}} = \frac{\sum_{t=1}^{\infty} \frac{t \cdot (1.02)^{t-1} \cdot 0.02}{1.02^t}}{\sum_{t=1}^{\infty} \frac{1}{1.02} t^{-1}} = \frac{\sum_{t=1}^{\infty} \frac{0.02}{1.02} t^{-1}}{\frac{1}{1.02} t^{-1}}
\]

Let \( \frac{0.02}{1.02} = 0.0197 = i^* \). Then the same as Q2.

\[
D = \frac{1 + \frac{1}{i^*}}{i^*} = 3.5
\]

4. (25%) A $1,000 bond with 8% annual coupons matures in 3 years at $1,100. It is purchased at a price to yield 9% effective. Determine the modified duration of the bond.

\[
\text{Numerator:} \quad 1 \cdot 80 \cdot 0.9 + 2 \cdot 80 \cdot 0.9^2 + 3 \cdot 1180 \cdot 0.9^3 = 2941.5928
\]

\[
\text{Denominator:} \quad 80 \cdot 0.9 + 80 \cdot 0.9^2 + 1180 \cdot 0.9^3 = 1051.9254
\]

\[
\text{or:} \quad PMT = 80 \quad N = 3 \quad i/y = 9 \quad FV = 1100
\]

\[
\text{CPT} \quad PV = -1051.9254 \quad \text{on BA-II}
\]

\[
D = \frac{2941.5928}{1051.9254} = 2.79644
\]

\[
MD = \frac{D}{1+i} = 2.566
\]