1. (25%) A 20-year loan of 20,000 may be repaid under the following two methods:
   
i) Amortization method with equal annual payments at an annual effective rate of 6.5%

   ii) Sinking fund method in which the lender receives an annual effective rate of 8% and the sinking fund earns an annual effective rate of j

Both methods require a payment of X to be made at the end of each year for 20 years.

Calculate j

(A) j < 6.5%
(B) 6.5% < j < 8.0%
(C) 8.0% < j < 10.0%
(D) 10.0% < j < 12.0%

(E) j > 12.0%

\[ 20,000 = P \cdot A_{20\%, 6.5\%} = P \cdot 1815.13 \]

\[ j \text{ interest } = 20,000 \times 8\% = 1600 \]

\[ P - 1600 = 215.13 \]

And annual 215.13 payments must accumulate to 20,000.

That is \[ 215.13 \cdot \frac{1-(1+j)^{-20}}{j} = 20,000 \]

\[ j = 12.0\% \]

2. (25%) Seth borrows X for four years at an annual effective interest rate of 8%, to be repaid with equal payments at the end of each year. The outstanding loan balance at the end of the third year is 559.12.

Calculate the principal repaid in the first payment.

(A) 444  (B) 454  (C) 464  (D) 474  (E) 484

\[ \text{Level payment} \]

\[ \text{principal repaid form a geometric progression.} \]

\[ \text{principal for 4th year: 559.12} \]

\[ \frac{559.12}{\sqrt[4]{1}} = 443.85 \]

\[ \text{Principal paid for 1st year} \]

Your answers: (Leave blank if you need no grading)

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3. (25%) Seth, Janice, and Lori each borrow 5000 for five years at a nominal interest rate of 12%, compounded semi-annually. Seth has interest accumulated over the five years and pays all the interest and principal in a lump sum at the end of five years. Janice pays interest at the end of every six-month period as it accrues and the principal at the end of five years. Lori repays her loan with 10 level payments at the end of every six-month period.

Calculate the total amount of interest paid on all three loans.

\[
\begin{align*}
\text{Seth:} & \quad 5000 \cdot (1 + \frac{0.12}{2})^{10} - 5000 = 3954.24 \\
\text{Janice:} & \quad 5000 \times 0.06 \times 10 = 3000.00 \\
\text{Lori:} & \quad \frac{5000 \times 10}{1 - 0.06} - 5000 = 6792.35 \times 10 - 5000
\end{align*}
\]

\[
\Sigma = 8747.64
\]

4. (25%) Ron is repaying a loan with payments of 1 at the end of each year for n years. The amount of interest paid in period t plus the amount of principal repaid in period t + 1 equals X.

Calculate X.

\[
\begin{align*}
(A) & \quad 1 + \frac{v^{t-1}}{i} & > & \quad i \cdot A^{n-t+1} = 1 - V^{n-t+1} \\
(B) & \quad 1 + \frac{v^{t-1}}{d} & \quad \text{the money you owe at } t-1 & \quad \text{equals to your future payment discounted to } (t-1) \\
(C) & \quad 1 + v^{t-1} & \quad \text{what you owe at } (t-1) \times i = \text{interest you paid at } t \\
(D) & \quad 1 + v^{-d} & \quad \text{year } (t+1) \ \text{principal repaid: } V^{n-t} \\
\end{align*}
\]

\[
\begin{align*}
\Sigma & = 1 - V^{n-t+1} + V^{n-t} = 1 - V^{n-t} (u-1) \\
& = 1 - V^{n-t} \left( \frac{i}{1+i} \right) = 1 - V^{n-t} \cdot d
\end{align*}
\]