MATH 370 Z
Quiz 5 – Yield Rates

1. (25%) You are given the following information about an investment account:

<table>
<thead>
<tr>
<th>Date</th>
<th>Value Immediately Before Deposit</th>
<th>Deposit</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 1</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>July 1</td>
<td>12</td>
<td>X</td>
</tr>
<tr>
<td>December 31</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

Over the year, the time-weighted return is 0%, and the dollar-weighted return is Y. Calculate Y.

\[
1 + \frac{r}{100} = \frac{12}{10} \cdot \frac{X}{12+X}
\]

\[
\text{From: } \frac{B - A - \sum C_k}{A + \sum C_k(1-t_k)} = 1
\]

\[
X = 60
\]

\[
\frac{X - 10 - X}{10 + 60 \cdot \frac{1}{2}} = -\frac{10}{40} = Y = -25\%
\]

2. (25%) On January 1, 1997, an investment account is worth 100,000. On April 1, 1997, the value has increased to 103,000 and 8,000 is withdrawn. On January 1, 1999, the account is worth 103,992. Assuming a dollar weighted method for 1997 and a time weighted method for 1998, the annual effective interest rate was equal to x for both 1997 and 1998.

Calculate x.

\[
\begin{align*}
\text{(A) } 6.0\% & \quad \text{(B) } 6.25\% \\
\text{(C) } 6.50\% & \quad \text{(D) } 6.75\% \\
\text{(E) } 7.0\% & \quad \text{Let } y = \text{account value at 1/1/98} \\
\text{At April 1, 3 month passed} & \\
\text{For 1997 (dollar weighted)} & \\
\text{For 1998 (time weighted)}
\end{align*}
\]

\[
x = \frac{y - 100,000 + 8,000}{100,000 - 8,000 \cdot (1 - \frac{3}{12})}
\]

\[
\frac{103,992}{y} = 1 + x
\]

\[
\Rightarrow x = 6.25\%
\]

Your answers: (Leave blank if you need no grading)

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>B</td>
<td></td>
</tr>
</tbody>
</table>
3. (25%) You are given the following information about the activity in two different investment accounts:

<table>
<thead>
<tr>
<th>Date</th>
<th>Account K</th>
<th></th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fund value before activity</td>
<td>Deposit</td>
<td>Withdrawal</td>
</tr>
<tr>
<td>January 1, 1999</td>
<td>100.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>July 1, 1999</td>
<td>125.0</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>October 1, 1999</td>
<td>110.0</td>
<td>2X</td>
<td></td>
</tr>
<tr>
<td>December 31, 1999</td>
<td>125.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date</th>
<th>Account L</th>
<th></th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fund value before activity</td>
<td>Deposit</td>
<td>Withdrawal</td>
</tr>
<tr>
<td>January 1, 1999</td>
<td>100.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>July 1, 1999</td>
<td>125.0</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>December 31, 1999</td>
<td>105.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

During 1999, the dollar-weighted (money-weighted) return for investment account K equals the time-weighted return for investment account L, which equals i. Calculate i.

(A) 10%  (B) 12%  (C) 15%  (D) 18%  (E) 20%

\[
\frac{25 - X}{100 - 0.5 \cdot 2X + 2X \cdot \frac{1}{4}} = \frac{25 - X}{100}
\]

\[
1+i = \frac{125}{100} \cdot \frac{105.8}{125 - X}
\]

\[
\Rightarrow \quad i = 15\%
\]

4. (25%) Esther invests 100 at the end of each year for 12 years at an annual effective interest rate of i. The interest payments are reinvested at an annual effective rate of 5%. The accumulated value at the end of 12 years is 1748.40. Calculate i.

(A) 6%  (B) 7%  (C) 8%  (D) 9%  (E) 10%

Original Account. 
Increase 100 every year.

Interest of Orig. Acc. 
(Also Inflow of re-invest acc.)
Increase 100i every year.

\[
\frac{1748.40 = 1200 + (100i) \cdot (1 + 0.05)^{11}}{1000} \Rightarrow \frac{748.40}{10.5} \cdot i \Rightarrow i = 7\%
\]