1. (25%) Jeff bought an increasing perpetuity-due with annual payments starting at 5 and increasing by 5 each year until the payment reaches 100. The payments remain at 100 thereafter. The annual effective interest rate is 7.5%. Determine the present value of this perpetuity.

\[ S_{0A} = \frac{5}{5 \%} \]

(A) 700  (B) 735  (C) 760  (D) 785  (E) 810

\[ PV = 5 \cdot (L_{\ddot{a}})_{20} + 100 \cdot \frac{1}{d} \cdot v^{20} \]

\[ \text{At } 0 \]

\[ PV = 5 \cdot (L_{\ddot{a}})_{20} + 100 \cdot \frac{1}{d} \cdot v^{20} \]

\[ = 785.40 \]

2. (25%) Jeff and Jason spend X dollars to purchase an annuity. Jeff buys a perpetuity-immediate, which makes annual payments of 30. Jason buys a 10-year annuity-immediate, also with annual payments. The first payment is 53, with each subsequent payment k% larger than the previous year’s payment. Both annuities use an annual effective interest rate of k%. Calculate k.

(A) 5  (B) 5.33  (C) 5.50  (D) 5.67  (E) 6

Note: \( k < \frac{100}{100} = 1 \)

\[ PV (Jeff) = \frac{30}{i} \]

\[ PV (Jason) = 53[(1+i)^{-1} + (1+i)^{-2} + \cdots + (1+i)^{-10}] = 53 \times 10 \times \frac{1}{1+i} = PV (Jeff) \]

\[ \therefore i = 6\% \]

**Your answers:** (Leave blank if you need no grading)

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3. (25%) A senior executive is offered a buyout package by his company that will pay him a monthly benefit for the next 20 years. Monthly benefits will remain constant within each of the 20 years. At the end of each 12-month period, the monthly benefits will be adjusted upwards to reflect the percentage increase in the CPI. You are given:
   (i) The first monthly benefit is $R$ and will be paid one month from today.
   (ii) The CPI increases 3.2% per year forever.
   At an annual effective rate of 6%, the buyout package has a value of $100,000. Calculate $R$.

(A) 517  (B) 538  (C) 540  (D) 548  (E) 563

Take "month" as time unit.

\[
(1 + \frac{1}{12})^{12} = 1 + \frac{i}{12}
\]

\[
i = 0.024967551
\]

\[
P_{V_{12}} = R \cdot A_{12}^{\frac{1}{12}} + \frac{1.032R \cdot A_{12}^{\frac{1}{12}}}{1 - (1.032)^{12}}
\]

\[
= 1.825064563 R = 100,000
\]

\[
R = \frac{547.93}{1.032} = 534.49
\]

4. (25%) Mary is to receive an annuity with 30 annual payments. The first payment of $1,000 is due immediately and each successive payment is 5% less than the payment for the preceding year. Interest 12% compounded monthly. Determine the present value of this annuity. [CAS 11/92 #20]

(A) Less than $6,400
(B) At least $6,400, but less than $6,500
(C) At least $6,500, but less than $6,600
(D) At least $6,600, but less than $6,700
(E) At least $6,700

\[
PV = 1000 \left[ 1 + \frac{0.95}{1.12} + \left( \frac{0.95}{1.12} \right)^2 + \cdots + \left( \frac{0.95}{1.12} \right)^{29} \right]
\]

\[
= 1000 \frac{1 - \left( \frac{0.95}{1.12} \right)^{30}}{1 - \frac{0.95}{1.12}}
\]

\[
= 6541 \quad \Rightarrow \text{Choose C}
\]

If used 12% as compound monthly rate, use \(i = (1 + \frac{12\%}{12})^{12} - 1\)

\[
P_{V'} = 1000 \frac{1 - \frac{0.95}{1.12683}}{0.95} = 6334.49
\]