1. (25%) A perpetuity costs 77.1 and makes annual payments at the end of the year. The perpetuity pays 1 at the end of year 2, 2 at the end of year 3, ..., n at the end of year (n+1). After year (n+1), the payments remain constant at n. The annual effective interest rate is 10.5%. Calculate n.

(A) 17 (B) 18 (C) 19 (D) 20 (E) 21

\[
\begin{align*}
77.1 &= \sqrt{\frac{2a}{n}} + n \cdot \sqrt{\frac{n+1}{i}} \\
2a &= n^2 - n \cdot \sqrt{n} + n \cdot \sqrt{n+1} \\
2a &= n^2 - n \cdot \frac{\sqrt{n} \cdot (n+1)}{i} \\
A &= 8.0955 \\
C &= \sqrt{n} \\
B &= n = 19
\end{align*}
\]

2. (25%) Payment of X are made at the beginning of each year for 20 years. These payments earn interest at the end of each year at an annual effective rate of 8%. The interest is immediately reinvested at an annual effective rate of 6%. At the end of 20 years, the accumulated value of the 20 payments and the reinvested interest is 5600. Calculate X.

(A) 121.67 (B) 123.56 (C) 125.72 (D) 127.18 (E) 128.50

\[
\begin{align*}
20X + 0.08X \cdot (\frac{1}{0.06} + \frac{20}{0.08 + 20}) &= 5600 \\
X &= 123.5 \text{ at } 5600.352.
\end{align*}
\]

Your answers: (Leave blank if you need no grading)

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3. (25%) A perpetuity-immediate pays 100 per year. Immediately after the fifth payment, the perpetuity is exchanged for a 25-year annuity-immediate that will pay X at the end of the first year. Each subsequent annual payment will be 8% greater than the preceding payment. The annual effective rate of interest is 8%.
Calculate X.

(A) 54 (B) 64 (C) 74 (D) 84 (E) 94

\[
\frac{100}{i} = X + 1.08 \cdot X \cdot \frac{1}{i} + 1.08^2 \cdot X \cdot \frac{1}{i}^2 + \ldots + 1.08^{25} \cdot X \cdot \frac{1}{i}^{25}
\]

\[
= 25 \cdot X \cdot \frac{1}{i}
\]

\[
X = \frac{4 \times 1.08}{0.08} = 54
\]

4. (25%) An insurance company has an obligation to pay the medical costs for a claimant. Average annual claims costs today are $5,000, and medical inflation is expected to be 7% per year. The claimant is expected to live an additional 20 years.
Claim payments are made at yearly intervals, with the first claim payment to be made one year from today.
Find the present value of the obligation if the annual interest rate is 5%.

(A) 88,932 (B) 102,514 (C) 114,611 (D) 122,634 (E) Cannot be determined

\[
PV = 5000 \cdot 1.07 \cdot 1.05^{-1} + 5000 \cdot 1.07^2 \cdot 1.05^{-2} + \ldots + 5000 \cdot 1.07^{20} \cdot 1.05^{-20}
\]

\[
= 5000 \left[ \frac{1.07}{1.05} + \frac{1.07^2}{1.05^2} + \ldots + \frac{1.07^{20}}{1.05^{20}} \right]
\]

\[
= 5000 \cdot \frac{1.07 - \frac{1.07^{20}}{1.05^{20}}}{1.05 - \frac{1.07}{1.05}} = 122,633.
\]