1. (25%) Let X be a continuous random variable with density function

\[ f(x) = \begin{cases} \frac{|x|}{10} & \text{for } -2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases} \]

Calculate the expected value of X.

(A) 1/5  
(B) 3/5  
(C) 1  
(D) 28/15  
(E) 12/5

2. (25%) An insurance company’s monthly claims are modeled by a continuous, positive random variable X, whose probability density function is proportional to \((1 + x)^{-4}\) where \(0 < x < \infty\). Determine the company’s expected monthly claims.

(A) 1/6  
(B) 1/3  
(C) 1/2  
(D) 1  
(E) 3

Your answers: (Leave blank if you need no grading)

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3. (25%) A random variable X has the cumulative distribution function:

\[
F(x) = \begin{cases} 
0 & \text{for } x < 1 \\
\frac{x^2 - 2x + 2}{2} & \text{for } 1 \leq x \leq 2 \\
1 & \text{for } x \geq 2 
\end{cases}
\]

Calculate the variance of X.

(A) 7/72  
(B) 1/8  
(C) 5/36  
(D) 4/3  
(E) 23/12

4. (25%) The warranty on a machine specifies that it will be replaced at failure or age 4, whichever occurs first. The machine’s age at failure, X, has density function

\[
f(x) = \begin{cases} 
\frac{1}{5} & \text{for } 0 < x < 5 \\
0 & \text{otherwise} 
\end{cases}
\]

Let Y be the age of the machine at the time of replacement. Determine the variance of Y.

(A) 1.3  
(B) 1.4  
(C) 1.7  
(D) 2.1  
(E) 7.5