1. (25%) A device runs until either of two components fails, at which point the device stops running. The joint density function of the lifetimes of the two components, both measured in hours, is

\[ f(x, y) = \frac{x + y}{27} \quad \text{for} \ 0 < x < 3 \text{ and } 0 < y < 3 \]

Calculate the probability that the device fails during its first hour of operation.

(A) 0.04   (B) 0.41   (C) 0.44   (D) 0.59   (E) 0.96

2. (25%) Two insurers provide bids on an insurance policy to a large company. The bids must be between 2000 and 2200. The company decides to accept the lower bid if the two bids differ by 20 or more. Otherwise, the company will consider the two bids further.

Assume that the two bids are independent and are both uniformly distributed on the interval from 2000 to 2200.

Determine the probability that the company considers the two bids further.

(A) 0.10   (B) 0.19   (C) 0.20   (D) 0.41   (E) 0.60

Your answers: (Leave blank if you need no grading)
3. (25%) A charity receives 2025 contributions. Contributions are assumed to be independent and identically distributed with mean 3125 and standard deviation 250.

Calculate the approximate 90th percentile for the distribution of the total contributions received.

(A) 6,328,000  (B) 6,338,000  (C) 6,343,000  (D) 6,784,000  (E) 6,977,000

4. (25%) Let X and Y be continuous random variables with joint density function

\[ f(x, y) = \begin{cases} 
15y & \text{for } x^2 \leq y \leq x \\
0 & \text{otherwise}
\end{cases} \]

Let \( g \) be the marginal density function of Y. Which of the following represents \( g \)?

(A) \( g(y) = \begin{cases} 
15y & \text{for } 0 < y < 1 \\
0 & \text{otherwise}
\end{cases} \)

(B) \( g(y) = \begin{cases} 
15y^2 & \text{for } x^2 < y < x \\
0 & \text{otherwise}
\end{cases} \)

(C) \( g(y) = \begin{cases} 
15y^2 & \text{for } 0 < y < 1 \\
0 & \text{otherwise}
\end{cases} \)

(D) \( g(y) = \begin{cases} 
15y^{3/2} (1 - y^{1/2}) & \text{for } x^2 < y < x \\
0 & \text{otherwise}
\end{cases} \)

(E) \( g(y) = \begin{cases} 
15y^{3/2} (1 - y^{1/2}) & \text{for } 0 < y < 1 \\
0 & \text{otherwise}
\end{cases} \)