MATH 370 X – Lecture 2
Conditional Probability & Independence

Today:

1. Conditional probability
2. Law of Total Probability
3. Bayes’ Rule

In Class Exercises:

1. A public health researcher examines the medical records of a group of 937 men who died in 1999 and discovers that 210 of the men died from causes related to heart disease. Moreover, 312 of the 937 men had at least one parent who suffered from heart disease, and, of these 312 men, 102 died from causes related to heart disease.

Determine the probability that a man randomly selected from this group died of causes related to heart disease, given that neither of his parents suffered from heart disease.

2. A doctor is studying the relationship between blood pressure and heartbeat abnormalities in her patients. She tests a random sample of her patients and notes their blood pressures (high, low, or normal) and their heartbeats (regular or irregular). She finds that:

(i) 14% have high blood pressure.
(ii) 22% have low blood pressure.
(iii) 15% have an irregular heartbeat.
(iv) Of those with an irregular heartbeat, one-third has high blood pressure.
(v) Of those with normal blood pressure, one-eighth has an irregular heartbeat.

What portion of the patients selected has a regular heartbeat and low blood pressure?
3. An insurance company insures a large number of homes. The insured value, $X$, of a randomly selected home is assumed to follow a distribution with density function:

$$f(x) = \begin{cases} 
3x^{-4} & \text{for } x \geq 1 \\
0 & \text{elsewhere} 
\end{cases}$$

Given that a randomly selected home is insured for at least 1.5, what is the probability that it is insured for less than 2?

4. An actuary studied the likelihood that different types of drivers would be involved in at least one collision during any one-year period. The results of the study are presented below:

<table>
<thead>
<tr>
<th>Type of driver</th>
<th>Percentage of all drivers</th>
<th>Probability of at least one collision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teen</td>
<td>8%</td>
<td>0.15</td>
</tr>
<tr>
<td>Young adult</td>
<td>16%</td>
<td>0.08</td>
</tr>
<tr>
<td>Midlife</td>
<td>45%</td>
<td>0.04</td>
</tr>
<tr>
<td>Senior</td>
<td>31%</td>
<td>0.05</td>
</tr>
<tr>
<td>Total</td>
<td>100%</td>
<td></td>
</tr>
</tbody>
</table>

Given that a driver has been involved in at least one collision in the past year, what is the probability that the driver is a young adult driver?