1. An auto insurance company insures an automobile worth 15000 for one year under a policy with a 1000 deductible. During the policy year there is a 0.04 chance of partial damage to the car and a 0.02 chance of a total loss of the car. If there is partial damage to the car, the amount $X$ of damage (in thousands) follows a distribution with a density function

$$f(x) = \begin{cases} 0.5003e^{-\frac{x}{2}}, & 0 < x < 15, \\ 0, & \text{otherwise}. \end{cases}$$

What is the expected claim payment?

(A) 320   (B) 328   (C) 352   (D) 380   (E) 540  
[05/01 #17]

2. The warranty on a machine specifies that it will be replaced at failure or age 4, whichever occurs first. The machine’s age at failure, $X$, has density function

$$f(x) = \begin{cases} \frac{1}{5}, & 0 < x < 5, \\ 0, & \text{otherwise}. \end{cases}$$

Let $Y$ be the age of the machine at the time of replacement. Determine the variance of $Y$.

(A) 1.3   (B) 1.4   (C) 1.7   (D) 2.1   (E) 7.5  
[05/01 #35]

3. $X$ is a continuous random variable with probability density function $f_x(x) = \frac{1}{2}x^2e^{-x}$ for $x > 0$ and $f_x(x) = 0$ otherwise. Find the second moment of $X$.

(A) $\frac{1}{2}$ (B) 1(C) 4(D) 12(E) 24

4. You are given a random variable $X$ whose moment generating function is $M_X(t) = \frac{6}{6-5t+t^2}$ for $t < 2$. Find the variance of this random variable.

(A) 3   (B) 1(C) $\frac{1}{2}$ (D) $\frac{1}{3}$ (E) $\frac{13}{36}$

5. If $X$ is a random variable with density function

$$f_x(x) = \begin{cases} 1.4e^{-2x} + 0.9e^{-3x}, & x \geq 0, \\ 0, & \text{elsewhere}, \end{cases}$$

Then $E(X) =$

(A) $\frac{9}{20}$ (B) $\frac{5}{6}$ (C) 1 (D) $\frac{230}{126}$ (E) $\frac{23}{10}$
6. Let the random variable $X$ have moment generating function $M_X(t) = \frac{1}{(1-t)^2}$ for $t < 1$. Find $E(X^3)$.
   
   (A) $-24(B)0(C)\frac{1}{4}(D)24(E)$ Cannot be determined from the given information

7. You are given the probability density function of a random variable $X$:
   
   \[ f_X(x) = \begin{cases} 
   2x, & 0 < x < 1, \\
   0, & otherwise.
   \end{cases} \]

   Find the difference between the third central moment and the second central moment of this random variable.
   
   (A) $\frac{1}{18}$ (B) $\frac{161}{135}$ (C) $\frac{17}{270}$ (D) $\frac{307}{270}$ (E) $\frac{307}{270}$

Answer: BCDEA DC