

# Math 540: Real Analysis I

## Spring 2008

### Basic Information

Instructor: Florin Boca

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Office: 359 Altgeld Hall

Phone: 244-9928

Lectures: MWF 1-1:50 pm, 443 Altgeld Hall

Office hours: Monday: 5-6 pm, Thursday: 5-6 pm, or by appointment

**Prerequisites:** Math 447 or equivalent.

You may contact the instructor for any queries or concerns.

### Course outline:

- I. **Abstract Lebesgue integral:**  $\sigma$ -algebras, topological spaces, Borel sets, Baire's category theorem and applications, measurable functions, the notion of measure, Lebesgue's integral and convergence theorems, comparison between Riemann and Lebesgue integration.
- II. **Lebesgue measure on  $\mathbb{R}^k$ :** Outer measure, Cantor functions, non-measurable sets, Lusin and Egorov's theorems, modes of convergence, the product measure, the Fubini-Tonelli theorem and applications, Lebesgue measure and integration in  $\mathbb{R}^k$ .
- III. **Differentiation and integration:** Bounded variation and absolute continuity, Lebesgue's differentiation theorems.
- IV. **The classical Banach spaces:** Convex functions and Jensen's inequality,  $L^p$  spaces: completeness, duals, embeddings.
- V. **Elementary Hilbert space theory and trigonometric series:** Inner products, convex sets, orthogonal decomposition, Riesz's representation theorem, orthonormal systems and bases in Hilbert spaces, Bessel's inequality and Parseval's identity, the Fejér kernel, the Riesz-Fischer theorem, completeness and convergence of trigonometric series.

**Text:** The classical books of Folland, Royden and Rudin are recommended but not required. Instructor's own notes will be made available.

**Grades:** Homework: 20%, two midterms: 40%, Comprehensive final: 40%.