CORRECTION ON
FACTORS OF TYPE III AND THE DISTRIBUTION OF PRIME NUMBERS

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At page 153, lines 5-14 should change to:

As in (2.8), we find constants $c_1, c_2 > 0$ such that for all $n \in \mathbb{N}$, $n \geq 2$,

$$c_1(n\log n)^{-1} e^{(\beta+2)n/3\beta t_0} < |B_n| < c_2(n\log n)^{-1} e^{(\beta+2)n/3\beta t_0}, \tag{2.12}$$

whence $\lim_n |B_n| = \infty$ and $\inf_n |B_{n+1}| / |B_n| > 0$. We may subsequently select N and nonempty subsets $B'_n \subset B_n$, $n \geq N$, such that $\inf_n |B'_n| / |B_n| > 0$ and $|B'_{2n}| = |B'_{2n+1}|$ for all $n \geq N$. We consider

$$B' = \bigcup_n B'_{2n+1} = \{p_m\}_m, \quad B'' = \bigcup_n B'_{2n} = \{q_m\}_m \subset B.$$

The elements p_j, q_k are all distinct and, from (2.11), $\lim_m p_m^\beta / q_m^\beta = e^{1/t_0} = \lambda^{-2}$. Moreover, (2.12) and $\lim_n \varepsilon_n = 0$ provide a constant $c_3 > 0$ such that

$$\begin{aligned}
\sum_m \frac{1}{q_m^\beta} &= \sum_{p\in B''} \frac{1}{p^\beta} = \sum_n \sum_{p\in B'_{2n}} \frac{1}{p^\beta} > \sum_n \frac{|B'_{2n}|}{e^{(\epsilon_n+2n+1/2)/t_0}} \\
&> c_3 \sum_n \frac{e^{2n(\beta+2)/3\beta t_0 - 2n/t_0}}{n\log n} = c_3 \sum_n \frac{e^{4(1-\beta)n/3\beta t_0}}{n\log n} \\
&\geq c_3 \sum_n \frac{1}{n\log n} = \infty,
\end{aligned}$$

which also entrains $\sum_m p_m^{-\beta} = \infty$. Summarizing, we have produced two sequences of distinct primes $\{p_m\}_m$ and $\{q_m\}_m$ in B such that

$$\lim_m \frac{p_m^\beta}{q_m^\beta} = \lambda^{-2} \quad \text{and} \quad \sum_m \frac{1}{p_m^\beta} = \infty = \sum_m \frac{1}{q_m^\beta}.$$