Math 347, HW # 5 (due Wed, Sep 26)

1)-6) Chapter 6: problems 6–11.

7) Find all $z \in \mathbb{C}$ for which $|z| - 2z = 3 - 4i$.

8) Solve in $\mathbb{C}$ the equation $(2 + i)z^2 - (4 - i)z + 1 = 0$.

9) For any $z_1, z_2 \in \mathbb{C}$ prove the triangle inequalities:

(i) $|z_1 + z_2| \leqslant |z_1| + |z_2|$.

(ii) $\big||z_1| - |z_2|\big| \leqslant |z_1 - z_2|$.

10) Let $z_1, z_2, z_3 \in \mathbb{C}$ such that $|z_1| = |z_2| = |z_3| = 1$ and $z_1 + z_2 + z_3 = 1$.

(i) Prove that $\dfrac{1}{z_1} + \dfrac{1}{z_2} + \dfrac{1}{z_3} = 1$.

(ii) Find z_1, z_2, z_3 knowing also that $z_1 z_2 z_3 = 1$.

11) Find a closed formula for the sums

$$S_1 = \sum_{k=1}^{n} \cos k\theta \quad \text{and} \quad S_2 = \sum_{k=1}^{n} \sin k\theta.$$

(Hint: Look at $S_1 + iS_2$ and at some (complex) geometric series.)

12) Let $z_0 \in \mathbb{C}$ with $|z_0| < 1$.

(i) Show that for every $z \in \mathbb{C}$ with $|z| \leqslant 1$ one has $\left|\dfrac{z - z_0}{1 - \overline{z_0}\, z}\right| \leqslant 1$.

(ii) Show that if $|z| = 1$, then $\left|\dfrac{z - z_0}{1 - \overline{z_0}\, z}\right| = 1$.