

**Math 595: Advanced Topics in Mathematics**  
**Title: Perron-Frobenius Operators** (CRN 64835, Section PFO)  
Spring 2017, Part A (Jan 17 - March 10)

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**Course description:** Perron-Frobenius operators provide a very powerful analytic tool in the study of the ergodic properties of piecewise continuous transformations  $T : I \rightarrow I$ , where  $I = [a, b]$ . For such a nonsingular transformation (i.e.,  $\lambda(A) \Rightarrow \lambda(T^{-1}A) = 0$  for every Borel set  $A \subset I$ , where  $\lambda$  is the Lebesgue measure on  $I$ ), the corresponding Perron-Frobenius operator  $P_T : L^1 \rightarrow L^1$  can be defined by the Radon-Nykodim derivative of the measure  $T_*\lambda$  with respect to  $\lambda$ , as follows:

$$\int_A P_T f d\lambda = \int_{T^{-1}A} f d\lambda, \quad \forall f \in L^1, \forall A \text{ Borel set.}$$

The spectral properties of this operator, or of related operators associated with  $T$  and defined on other spaces of functions, provide a wealth of information about the ergodic properties of the original transformation, such as existence of absolutely continuous invariant measures, ergodicity, mixing, or distribution of periodic points.

This one-semester course will provide an introduction to this topic, focusing mainly on applications to transformations of number theoretical nature arising from continued fractions, such as Gauss and Farey type transformations.

**Prerequisite:** Math 540 and Math 541, or approval of instructor.  
**Textbook:** There is no required textbook.