Math 542: Complex Variables I
Spring 2012

Instructor: Florin P. Boca
Office: 362 Altgeld Hall
Phone: 244-9928
E-mail: fboca@illinois.edu

Lectures: MWF 10:00-10:50 am, 143 Henry Administration Building
Office hours: Monday: 4-5 pm, Thursday: 5-6 pm.

Course description: This is the core graduate course in Complex Analysis, covering the following topics:

- *Complex number system.* Basic definitions and properties; topology of the complex plane; connectedness, domains. Riemann sphere, stereographic projection.
- *Differentiability.* Basic definitions and properties; Cauchy-Riemann equations, analytic functions.
- *Elementary functions.* Fundamental algebraic, analytic, and geometric properties. Basic conformal mappings.
- *Contour integration.* Basic definitions and properties; the local Cauchy theory, the Cauchy integral theorem and integral formula for a disk; integrals of Cauchy type; consequences.
- *Sequences and series.* Uniform convergence; power series, radius of convergence; Taylor series.
- *The local theory.* Zeros, the identity theorem, Liouville's theorem, etc. Maximum modulus theorem, Schwarz's Lemma.
- *Laurent series.* Classification of isolated singular points; Riemann's theorem, the Casorati-Weierstrass theorem.
- *Residue theory.* The residue theorem, evaluation of certain improper real integrals; argument principle, Rouche's theorem, the local mapping theorem.
- *The global theory.* Winding number, general Cauchy theorem and integral formula; simply connected domains.
- *Uniform convergence on compacta.* Ascoli-Arzela theorem, normal families, theorems of Montel and Hurwitz, the Riemann mapping theorem.
- *Infinite products.* Weierstrass factorization theorem. Runge's theorem. Applications.
- *Harmonic functions.* Definition and basic properties; Laplace's equation; analytic completion on a simply connected region; the Dirichlet problem for the disk; Poisson integral formula.

Prerequisite: MATH 446 and MATH 447, or MATH 448, or equivalent.

Textbook: *T. W. Gamelin, Complex Analysis, Springer, 2001.* Other recommended books: B. Palka, An Introduction to Complex Function Theory, Springer, 1990, and W. Rudin, Real and Complex Analysis, 3rd edition, McGraw-Hill.

Grading policy: Comprehensive final exam: 45%; Two midterm exams: 2x20 = 40%; Homework: 15%.