

Math 540: Real Analysis
Fall 2017

Instructor: Florin P. Boca

Office: 371 Altgeld Hall

E-mail: fboca@illinois.edu

Lectures: MWF 1:00-1:50, 140 Burrill Hall

Office hours: Monday 3:30-4:30, Thursday 5:00-6:00

Course description: This is the core graduate course in Real Analysis, covering the following topics:

- *Measures on the line.*
Abstract measure theory, outer measure, Lebesgue measure on the real line, measurable sets, Borel sets, Cantor sets and functions, non-measurable sets, Baire's category theorem.
- *Measurable functions.*
Structure of measurable sets, approximation of measurable functions by simple functions, Littlewood's three principles, Egorov and Lusin's theorems.
- *Integration.*
Lebesgue theory of integration, convergence theorems (Monotone Convergence, Fatou's Lemma, little Fubini, Dominated Convergence), comparison of the Riemann and Lebesgue integrals, modes of convergence, approximation of integrable functions by continuous functions, Fubini's theorem for the plane, product measures, the general Fubini-Tonelli theorem, applications to probability, the convolution product.
- *Differentiability.*
Functions of bounded variation (structure and differentiability), absolutely continuous functions, maximal functions, fundamental theorem of calculus, the Radon-Nikodym theorem.
- *L^p spaces on intervals and ℓ^p spaces.*
Jensen's inequality, Hölder and Minkowski's inequalities, class of L^p functions, completeness, duals of L^p ; spaces, inclusions of L^p spaces.
- *Hilbert spaces and Fourier series.*
Elementary Hilbert space theory, orthogonal projections, Riesz representation theorem, Bessel's inequality, Riemann-Lebesgue lemma, Parseval's identity, completeness of trigonometric spaces.

Prerequisite: MATH 447 or equivalent.

Textbook: There is no required textbook. Recommended textbooks include:

- Gerald B. Folland, Real Analysis. Modern Techniques and Their Applications.
- Peter Loeb, Real Analysis.
- Walter Rudin, Real and Complex Analysis.

Grading policy: Comprehensive final exam: 40%; Midterm exams: $2 \times 20 = 40\%$; Homework and quizzes: 20%.

Exam calendar: Midterm 1: Mon Oct 2; Midterm 2: Fri Nov 10

Final exam: 8-11 am, Wednesday Dec 20, 140 Burrill Hall.