Print name

Math 453, Hour Exam # 2

October 24, 2013

No calculators or other e-devices and no books or notes are allowed.

Solve ANY FOUR problems. Mark the numbers of these problems on the front page. Extra-credit will be given for substantial work on the remaining problem.

Show complete work to qualify for full credit.

Each of the marked four problems is worth 10 points. Extra-credit is worth 5 points.

[1] Find the least nonnegative residue modulo $m = 11$ of $n = \dfrac{31!}{22!}$.

[2] Find the least nonnegative residue modulo $m = 19$ of $n = 3^{10^6}$.

[3] Show that $30|(n^9 - n)$ for all positive integers n.

[4] Prove that the following three identities hold for every $n \in \mathbb{N}$:

(a) (3 pts.) $\displaystyle\sum_{d|n} \frac{\mu(d)}{d} = \frac{\varphi(n)}{n}$.

(b) (3 pts.) $\displaystyle\sum_{d|n} \varphi(d) = n$.

(c) (4 pts.) $\displaystyle\sum_{d=1}^{n} \varphi(d)\left[\frac{n}{d}\right] = \frac{n(n+1)}{2}$.

[5] (2pts.) (a) Given two arithmetic functions f_1 and f_2, give the definition of their convolution product $f_1 * f_2$.

(b) (3 pts.) State (without proofs) five properties of the convolution product of multiplicative arithmetic functions.

(c) (2 pts.) Compute $\mu * \mathbf{1}$ and $\mathbf{1} * \mathbf{1}$ where $\mathbf{1}(n) = 1$ for all $n \in \mathbb{N}$.

(d) (3 pts.) Prove the following identity:

$$\sum_{n=1}^{\infty} \frac{\nu(n)}{n^s} = \zeta(s)^2 \qquad \text{for all } s > 1.$$