Print name

Math 453, Hour Exam # 1

September 27, 2013

No calculators or other e-devices and no books or notes are allowed.
Solve ANY FOUR problems. Show complete work to qualify for full credit.
Problems carry the same weight.

[1] Prove that there are infinitely many prime numbers.

[2] (i) Let $a, b, q, r \in \mathbb{Z}$, $b \neq 0$, such that $a = bq + r$. Prove that $(a, b) = (b, r)$.
(ii) Does a multiplicative inverse of $n = 14$ modulo 79 exist? Explain why and compute one in the affirmative situation.

[3] Find the prime factorization of $n = 10^6 - 1$.

[4] (a) Suppose that $a, b, m \in \mathbb{Z}$, $m \geq 1$, satisfy the following three conditions:

$$\text{(i) } a^2 \equiv b^2 \pmod{m} \qquad \text{(ii) } a^3 \equiv b^3 \pmod{m} \qquad \text{(iii) } (b, m) = 1.$$

Prove that $a \equiv b \pmod{m}$.
(b) Prove by mathematical induction that if $n \in \mathbb{N}$, then

$$4^n \equiv 1 + 3n \pmod{9}.$$

[5] Find the general solution $x \in \mathbb{Z}$ of the following system of congruences:

$$\begin{cases} 2x \equiv 1 \pmod{3} \\ 3x \equiv 2 \pmod{5} \\ 5x \equiv 4 \pmod{7}. \end{cases}$$

Total score: $40 = 4 \times 10$ **points.**